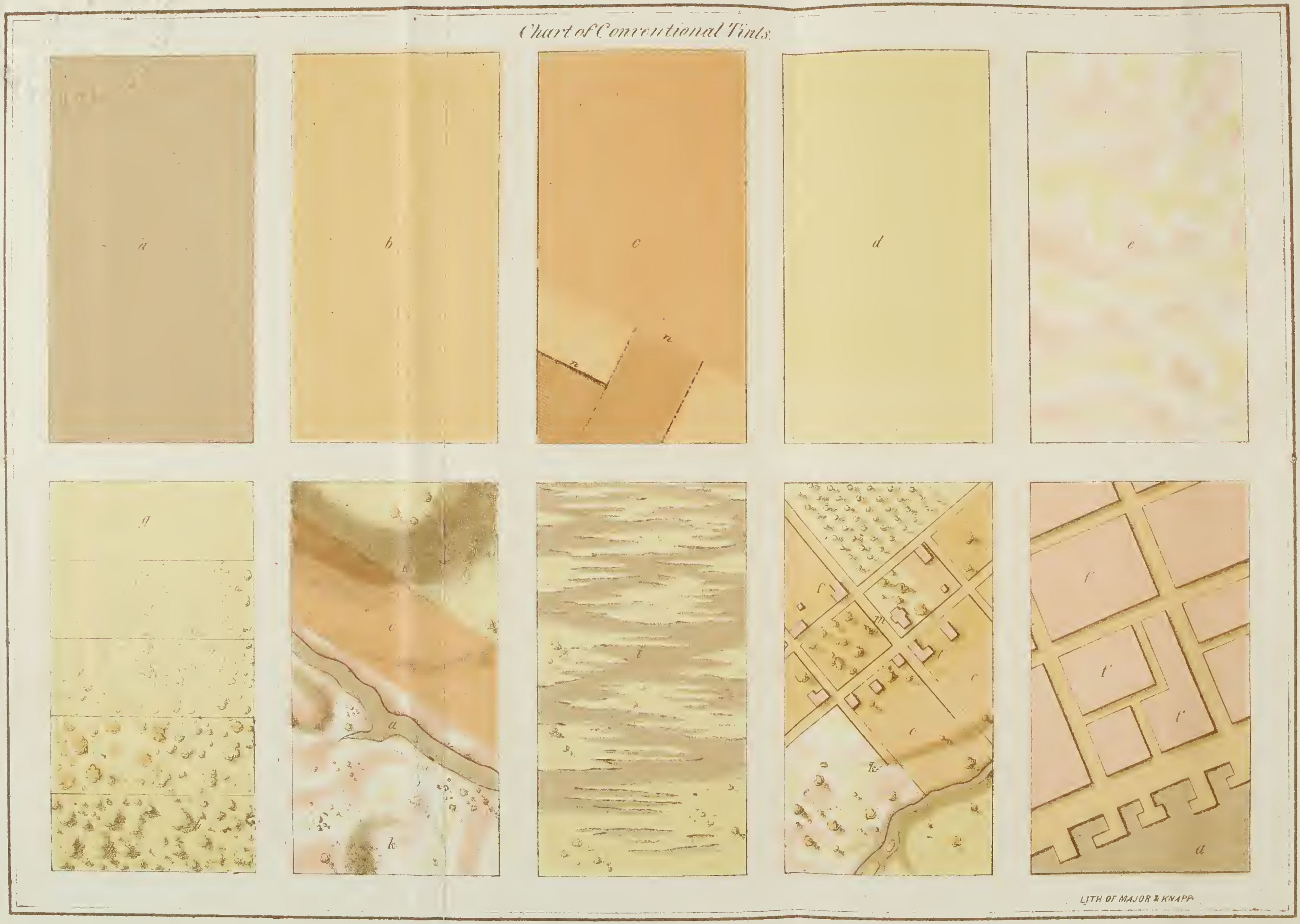
Fig. 36.



A MANUAL

OF

TOPOGRAPHICAL DRAWING.

BY

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NEW EDITION WITH ADDITIONS AND EXTRA PLATES.

NEW YORK:
JOHN WILEY & SON,
15 Astor Place.
1882.

TO

ROBERT W. WEIR, N.A.,

PROFESSOR OF DRAWING

IN THE

UNITED STATES MILITARY ACADEMY,

West Point, N. Y.,

THIS MANUAL IS

MOST RESPECTFULLY INSCRIBED

PREFACE.

THE following pages have been prepared to meet, and in some degree to supply, the demand for practical instruction in Topographical Drawing. The great activity which prevails in regard to Internal Improvements, is constantly calling into the field numbers of young Engineers; and already many instructive works have been addressed to them on almost every detail connected with their profession, except map-making.

The design of this little manual is, to be a practical assistant and office companion, to be consulted on all matters connected with Topographical Drawing, from the first sketch of a preliminary survey, to the complete map. Its scope is limited to field and office drawing, and nothing else is treated of, but what relates to, or is illustrative of, those departments of Topography.

With regard to the explanatory figures, the greater part of them are autographic, and, of course, inferior in point of execution to the fine engravings that usually accompany such works as this. The author would plead that they are intended rather as illustrations of methods than specimens of style; and that the student is more familiarly and intelligibly addressed by means of the pen and ink, than by the unapproachable perfection of copper-plate.

The conventional signs everywhere in use, are those here employed and explained, but all arbitrary and unnecessary multiplication of them has been studiously avoided.

WEST POINT, N. Y.
August, 1854.

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INTRODUCTION.

1. When a topographical drawing is to be made with the pen, upon Demy or Royal paper, select the smooth side of it, and draw the rectangle intended to contain the map, in the middle of the sheet. To do this, find the intersection of the two diagonals of the paper, by laying a rule from corner to corner, and drawing light pencil lines near the middle. This intersection will be the middle of the sheet; with which the centre of the drawing must coincide. Draw through this central point a line parallel to the lower edge of the sheet, then perpendicular to this, and through the same central point, another line. The former of these central lines will give the *direction* of the upper and lower bases of the drawing, and the latter that of the upright sides. Lay off, from the central point, to the right and left, on the horizontal central line, distances equal to half the base of the required rectangle, and through the points thus found, draw lines parallel to the upright central line: these will be the indefinite upright sides. Then through two points on the upright central line, at distances above and below the centre equal to half the altitude of the required rectangle, draw lines parallel to the horizontal central line, and these will complete the rectangle and form its upper and lower bases.

2. If the drawing is not a square, the longest line of its margin must be laid in the direction of the longest edge of the paper.

3. When a drawing is to be finished with the pen, it should be always borne in mind that the lead-pencil is used only as a guide, or preparation for the pen, and that all pencil lines, without exception, must be drawn *very lightly*, with a moderately hard pencil, finely pointed, which being drawn two or three times over a line with a very slight pressure, will produce a mark which may be seen very distinctly, and be easily rubbed out afterwards.

4. Ruled pencil lines should be drawn a little beyond their exact length, for in going over them afterwards with a pen, their intersections can be more readily distinguished by means of these projecting ends of the lines.

5. In copying from a drawing, it is usual, in order to facilitate the getting in of the outline, to draw a number of lines of some simple arrangement upon the model, and to do the same with the rectangle in which the copy is to be made. The easiest method is to divide the drawing into squares, whose sides are in direction parallel to the margin lines respectively, and in length some multiple of the shorter side of the drawing, such as $\frac{1}{8}$, $\frac{1}{10}$, or $\frac{1}{16}$. It is evident, however, that any kind or number of lines drawn upon the model will answer the same purpose, provided the proposed copy is treated in exactly the same manner. Having, then, these similar systems of lines, it will be easy to cause the outlines in the copy to pass through squares corresponding to those of the model. This process at once suggests the method of enlarging or reducing a drawing by increasing or diminishing the sides of the corresponding squares. In comparing the proportions of similar drawings, linear measure is always used; e. g., a drawing is said to be twice the size of another when it is twice as high and twice as wide, though it contains four times the surface. The squares must be drawn in pencil, and lightly, as they would disfigure the drawing if they could not be entirely removed.

6. Indian Ink is used in finishing pen drawings. It should be of the best quality, which insures its quick and perfect mixture with water, and should be rubbed up perfectly black, in a small plate, as pale ink makes the boldest drawings look weak. To test its blackness when mixed, take some in a pen, make a pretty broad mark with it upon white drawing paper, and wait until it dries, when it will display its true strength. After it has become black, it is ready for use, and any further mixing will make it viscid.

7. The steel pens now so generally used are perhaps the best for drawing, and tend to produce, by their superior durability, an evenness of style. The pen should be not too elastic, nor should it be easily turned from its direction by an increase of pressure upon it. Some draftsmen prefer quill pens, which, when of a good quality, well made, and frequently renewed, are certainly unobjectionable.

8. Pencils, as before remarked, should be moderately hard for line drawing. Faber's No. 3, or Wolff's HHH are of the proper hardness. They should be pointed by rubbing them on a piece of fine sand paper. (Par. 3.)

9. In general, all lines drawn by hand (that is, without ruling), are more

easily drawn *towards the body;* and with this view, a topographical drawing should be turned on the table in any way that will facilitate that manner of drawing. Ruled lines are more conveniently drawn from left to right (always along the upper edge of the rule), and for that purpose also the drawing should be turned in any direction. But in copying the outlines by the eye, both the model and the copy must be placed upright before the draftsman, so that the line he is engaged in drawing may be really parallel to the one he is copying.

10. *No line that is meant to be a straight line* should be drawn *by hand.* Right lines, whether in pen or pencil, must invariably be ruled, no matter how short they may be; and if in ink, should be drawn with the right-line pen. Nor should a right angle ever be *guessed at.* All square corners, whatever be the shortness of the lines forming them, must be constructed with the proper instrument. Parallel lines also, no matter how short, must be *constructed.*

11. In making or copying a drawing, begin with the principal lines in it; for example, if a broad stream or an extended sheet of water be represented, begin with that; then proceed to the roads, and smaller streams. Prepare everything completely in pencil, before taking up the pen to finish; for in doing so, the progress of the work is more satisfactory and apparent.

12. In copying lines which are so close together as that many of them are contained in a single square (which is sometimes the case with horizontal curves), that square can be subdivided on the model and on the copy, by joining the middle points of the opposite sides, so that one square will be made into four. Or, only alternate curves may be studied and drawn, and the intermediate ones can afterwards be easily introduced.

12. Let the drawing be kept clean. A piece of thin paper should be constantly interposed between its face and the draftsman's hands. The ink plate should be kept on one side, and never in front of the drawing Preserve the paper on which the drawing is in progress from being bruised: it should never hang over the edge of the table where the body or arms can press upon and break it.

TOPOGRAPHICAL DRAWING.

1. TOPOGRAPHY is the art of describing the minute features of the earth's surface. Topographical Drawing consists in representing, by lines, or some other conventional expressive means, the exact shape and figure of the ground in a particular locality, as well as the dimensions and positions of all objects situated upon its surface. Two classes of objects present themselves for description; the one *natural*, including, 1st.—mountains, of every extent, their slopes, their rocky sides, their gorges and valleys, and in general, every inequality in the surface of the ground; 2d.—bodies of water, as the sea, rivers, brooks, lakes, ponds, and marshes; and 3d.—all natural productions or conditions of the ground, such as forests, heath, meadows, sand, &c. The other class comprises artificial works, such as buildings, inclosures, cultivation, roads, &c. Of this latter class, buildings may be divided according to their importance, as churches, country-seats, farm houses, &c. Inclosures may be variously represented as ditches, hedges, walls, or fences. The different kinds of cultivation need not to be discriminated; but where a distinction is desired, it is better to display it by lettering the ground neatly. Roads, or communications, are distinguished as turnpikes, railroads, canals, cross-roads, foot-paths, fords, &c.

2. Every topographical drawing addresses itself to the eye as if the spectator were situated above, and looking down equally upon every part of it. In representing therefore upon such a drawing the relative positions and the dimensions of objects, accurate measurements, according to some assumed scale, are used, and distances are laid off, as in any other plan-drawing. But in expressing the nature of objects, many of

them not being bounded by mathematical or regular lines, recourse is had to certain conventional means, universally agreed upon among draftsmen. In some instances, the signs thus used are made to resemble, in some degree, the objects for which they stand, as in the case of forests, rocks, meadows, &c. In others, they are purely conventional, as in the case of hills, water, marsh, &c. The pen, the brush, and the pencil, by means of lines or colors, offer facilities for topographical drawing, which it is proposed to consider in the order in which they are mentioned.

3. The first characteristic to which we naturally direct our attention in regard to the topography of a locality, is the variation in the surface of the ground, with reference to hill, valley, and plain. The two general systems of delineating with the pen these important features, will first be noticed. These are called, respectively, the *horizontal* and the *vertical* system.

4. First. *The Horizontal System.*—This consists in intersecting the inequalities of the ground by a series of horizontal planes, at equal vertical distances apart, and "projecting" upon the map the curves in which these planes intersect the surface. To explain this process by a familiar illustration, let us suppose a hill, rising out of the water, as in *Fig.* 1, where such a hill is represented *in profile*—AB being the water-surface. If we should walk completely around the base of this hill, exactly along its water-mark, we should follow a perfectly *level*, or *horizontal curve*, for it is formed on the hill side by the horizontal surface of the water. This water line may then be called, a curve "*cut out of the hill by a horizontal plane*," and, as such, we may measure its dimensions, determine its proportions, and draw, or "project" it on our plan.

5. Suppose, now, the water *to rise* one foot. A new curve will be defined on the hill-side, in a manner similar to the first, and at a vertical distance of one foot above it at every point. This new curve will possess properties similar to the first one, and may, like it, be determined and projected. CD is the plane of the second curve. In the same manner, the planes of other curves, at the same vertical distance apart, may be conceived, and the curves measured and drawn, as EF, GH, IK, and LM.

6. Let the curves now be projected upon a horizontal plane; that is, suppose the eye to be placed above the hill, so as to look directly down upon every point of its surface. The curves will then be drawn, as in *Fig.* 2 (the shading lines excepted).

7. In this topographical plan, each of the horizontal curves gives us, throughout its length, an exact idea of the shape of the ground. If we know (as is always known from the other parts of the map) that the *inner*, or smallest curve, is elevated above the others, then we have the representation of a *hill.* If, on the contrary, the *outer* curve is elevated above all the others, then our drawing represents a *hollow.*

8. In the spaces between these horizontal curves, or sections, we are necessarily left in ignorance of the precise form of the ground. But our knowledge will increase with the number of curves we have; and if they could be drawn at very small vertical distances from each other, or nearly in contact, we should have an almost perfect representation of the slope. This, however, is neither practicable nor necessary, for when we have obtained such a number of sections as will furnish a knowledge of the ground sufficient for the purposes required (and this number may be increased according to the requirements of the case), it is allowable, and customary, to consider the ground between any two sections as sloping *uniformly.* The space between any two of these curves is called a *horizontal zone.* This zone may be considered as generated by a straight line, placed in the direction of the slope of the zone, and touching *both* of the curves, and which, being kept normal* (perpendicular) to the *upper* one of the pair, is moved around the hill, fulfilling constantly the above conditions, until it returns to the point whence it set out. The successive positions of this line are the elements, constituting the surface of the zone.

9. A comparison (*Figs.* 1 *and* 2) of the curves with the corresponding profile, will show that where the hill is steep, the horizontal sections are projected close together, and that where the hill is not so steep, they are projected at a greater distance from each other; and in proportion to the

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* A normal to a curve is a right line, which is perpendicular to a tangent to the curve, at the point of contact.

proximity or remoteness of the sections, is the steepness or gentleness of the declivity. This proportion will always exhibit the *relative* degrees of the slope at any part of the surface.

10. In order to find the actual inclination of a zone at any place, find the element at that place by drawing a normal to the upper curve of the zone. Construct, with the normal line *m n* (*Fig.* 2) so found, as a base, and with the known vertical distance between the sections as an upright, a right angled triangle; its hypothenuse will be equal to the true line of the slope, and the acute angle at the base will be equal to the angle of inclination required.

This triangle is seen at AC*a* (*Fig.* 1), and the angle CA*a* is the angle of inclination. Or construct, for all cases, a *scale* of inclinations, as follows:—Draw the lines AO and OB (*Fig.* 3), forming a right angle at O. Lay off with a protractor, the lines O 5, O 10, O 15, O 20, &c., making, with OB, angles of 5°, 10°, 15°, 20°, &c., successively up to 45°. To use this scale, draw the line CD parallel to OB, and at a distance above it equal, according to the scale of the map, to the vertical distance between the horizontal sections. Having, as aforesaid, found the normal at any point, take its length in the dividers (or compasses), set one foot of the dividers at C, and the other towards D, on the line CD, and observe its position with regard to the intersections of CD with any of the lines O 5, O 10, &c. For example, should the length of the normal be C*e*, the inclination is 5°; should it be C*g*, it is 10°; should its extremity fall at *f*, midway between *e* and *g*, the inclination is 7½°. If it fall at a point not easily determined as to its position, draw a line from O through it, and measure, with a protractor, the angle formed with OB. This scale may be made more exact, by laying off angles of less than 5°.

11. To show how these horizontal sections contribute to the knowledge of forms, *Figs.* 4, 5, and 6, are plans and profiles of the pyramid, cone, and hemisphere. The upper portion of *Fig.* 4 is an *elevation*—that is, a vertical view, or "projection" of a square, right pyramid. It exhibits the *vertical* dimensions. Suppose this pyramid to be three inches in height, and to be made of wood, or some soft material. Drill a fine hole from the apex to the middle of the base; it will pass through the central line, or axis, of the figure, and will be perpendicu-

lar to the base. Suppose, now, that we saw or cut the pyramid into six slices, $\frac{1}{2}$ inch thick each, by passing the saw parallel to the base at each cut. Set up the pyramid again, pass a stiff wire or needle through the drilled axis, and place it upon a sheet of paper on a horizontal table, so that one of the lines forming the base shall be parallel to, or coincident with, a right line drawn on the paper. Press the wire-axis, so as to mark the centre of the base, and draw a pencil line around the base of the body. This will be the lower outline of the undermost zone. Remove now, the lowest slice, and let the next one above come down upon the paper, keeping the axis in the same place, and the sides of the base *in the same direction* as before. Draw a line around this new base with a pencil, and it will give the upper outline of the undermost zone, or, which is the same thing, the lower outline of the zone next to the lowest one. By repeating this operation in the same way, we shall obtain the upper and lower outlines of all the six zones of the figure. The upper outline of the upper zone is the apex, and will be represented by one point, in the middle of the base. The sections of such a pyramid will thus be found to be a series of concentric squares, whose vertical distance from each other is the assumed thickness of the slices into which it has been divided, and whose horizontal distance apart is the base of a right-angled triangle, whose perpendicular and hypothenuse are respectively the vertical thickness of the zone, and the length of the normal, or true slope of the zone. (See *Fig.* 4, at *a*, *b*.)

The same method of demonstration, and the same remarks, will apply to the cases of the cone and hemisphere; and an examination of the figures (5 *and* 6) will show the manner of drawing, and connecting the vertical and horizontal projections (or the *elevations* and *plans*) of those bodies, when treated in the same way as the pyramid we have been considering.

The sections of the cone (a right cone with circular base) and of the hemisphere are concentric circles. In the case of the cone they are equidistant, and their horizontal distance is found as in the pyramid. *This equidistance of the sections indicates a uniform declivity.* In the hemisphere the upper sections are more removed from each other than the lower, and the horizontal distance between them diminishes rapidly as we

approach the lower ones, which *indicates an increase in the declivity* (*Fig.* 6). The eye should be familiarized with this kind of projection, so that a distinct idea of the form of any object may be received or conveyed by means of sections cut out by equidistant horizontal planes.

12. We have thus far considered a hill as given only by such a number of sections as could, or needed to be accurately determined by survey or otherwise, assuming the slope between these determinate sections to be *uniform*. But we can now fill up this skeleton plan, and draw any desired number of curves in each zone, by dividing and sub-dividing the normals by medial sections. If we draw through the middle points of the normals of any zone a medial line, we shall have (by the principle—*Par.* 11—that equidistant sections indicate uniform slopes) a new curve cut out of the surface by a medial horizontal plane. Dividing medially these secondary zones, we shall obtain two more auxiliary curves, and so on for any number.

It is upon this principle that drawings are filled up or shaded, in the horizontal system. When we have determined and projected a sufficient number of curves, so that the assumed uniformity of slope between them does not differ essentially from the truth, we can proceed to fill in, without constructing them, as many curves as it may be desirable to have in the spaces between the determinate ones. And if we keep always the same number of shading lines within each pair of curves, they will, by their proximity or remoteness, as the width of the zone diminishes or increases, serve to show the relative steepness of the slope. The darker shade produced by their drawing closer together, and the lighter color caused by their separation, will contribute to the same effect. And it is from this (in connexion with vertical illumination, which will be referred to in explaining the vertical system) that has been derived the conventional idea that a dark color represents a steep slope, a lighter color a more gentle one, and perfect white a level. See *Fig.* 7 for an example of this method of finishing hills.

13. Practical Directions and Remarks on Drawing a Hill according to the Horizontal System.

Having prepared the outlines of the drawing in light pencil lines, including the skeleton curves of all the hills or hollows,

proceed to finish with the pen and ink. In commencing the shading lines, place the drawing on the table so that the summit of the hill shall be towards the left hand (see *Fig.* 2). Then draw (towards the body) as many lines within the highest pair of curves (say six, more or less, according to the large or small scale of the map), as you intend to put in each horizontal zone. Thus, if the determinate curves are constructed at a vertical distance of one foot apart, our auxiliary ones will be about two inches apart vertically. If the scale is smaller, and the determinate curves are drawn six feet apart, our shading lines will have a vertical distance of about one foot. Draw the lines with firmness, and let them have a length varying from about $\frac{1}{4}$ of an inch to about $\frac{3}{4}$ of an inch, according to the width of the zone, that is, according to the greater or less degree of declivity (*Fig.* 2, at *o* and *p*). Where the hill is steep, the lines are heavy and short; where it is less steep, they are longer and lighter, and in approximating the level they must be drawn as fine and clean as possible. Let them divide equally the space between the curves. Go all around the hill in this manner, in each zone, before commencing the one next below it, turning the drawing on the table so as to draw always towards the body, increasing or diminishing the distance between the lines as the width of the zone varies, so that they just fill the zone evenly; and finish by joining them together where they began. Proceed in like manner with the second zone, and so on to the bottom of the hill. Inasmuch as we have the form of the hill accurately defined in pencil by the skeleton of curves, it is not absolutely necessary that, in shading, the accessory curves should be rigorously *continuous.* A slight variation in their position at the joints, provided they do not wander out of their zone, imparts a degree of freeness to the style. Be careful to connect the sets of lines together, end to end, so that the groups shall not be separated by a vacant space (*Fig.* 2, at *r*), or be overlapped by thrusting the lines of one group between those of another (*Fig.* 2, at *s*). Endeavor to obtain a clear, even tint. Do not let the junctions of the groups in the different zones form continuous lines down the hill (*Fig.* 2, *d*, *d*), but let them "break joints," by frequently bringing a junction opposite to the middle of the group in the zone above (*Fig.* 2, at *e*, *e*, *e*).

14. When a drawing of a hill is finished according to this system, and the pencil lines removed, the zones formed by the determinate curves are no longer distinguishable, and the means of ascertaining actual heights and inclinations are lost. This difficulty may be obviated either by marking the determinate curves by light lines of *red*, and numbering them according to their vertical distance above or below a certain assumed level, and from each other; or by writing upon the map a statement of the vertical thickness of a zone, and how many auxiliary sections are contained in it.

15. Whenever this horizontal system is used in representing slopes, the conventional sign for bodies of water consists only of a narrow strip of tint, or shade, produced by short lines drawn *parallel to the base of the drawing.* Draw these lines towards the body, and from the shore outwards, having first defined the width of the strip by means of a fine pencil line (*Fig.* 7 at A).

16. The other method of representing hills, which is called the Vertical System, is now to be considered. It consists in expressing the inclination of a hill-side, by drawing its lines of *greatest descent.* The two required elements are both to be exhibited in these lines, viz. the *direction* and *degree* of the slope. The *direction* of any line of greatest descent (which is the true direction of the slope or inclination), is obtained by considering the horizontal sections of the surface. Let us suppose a hill (*Fig.* 2) given by its curves. If, from the summit of this hill, we suppose water to flow, or a round mass or body to be rolled, it will evidently, under the influence of gravitation, seek the lowest point, and that by the *shortest line.* This path, so described, will be the "*line of greatest descent.*" It is further to be observed, that this line will be found constantly perpendicular (normal) to the horizontal sections. For any deviation from this direction (*Fig.* 2, *f*, *g*) displays a spiral tendency, or a tendency to move *around* the hill, which cannot be imparted by gravitation.* Having then the horizontal sections given, we can always draw, perpendicular to them, any num-

* Any deviation from a direction perpendicular to the horizontal lines is an approximation towards a line of *no* descent, and in so far, a departure from a line of *greatest* descent.

ber of lines of greatest descent, thereby obtaining a complete knowledge of the *direction* in which the ground slopes (*Fig.* 15).

17. The other, equally important, element of the inclination, viz. its *degree* of declivity, is expressed by means entirely conventional. Two methods have been adopted for this purpose: one depending upon the principle of vertical illumination, in which the maximum light is reflected upwards to the eye by a horizontal surface, and a minimum by a surface inclined 45° to the horizon. The latter, being the steepest slope at which earth will stand, is taken as the limit of least illumination. This is the English and German convention, and lays more stress on the different proportions of black to white in indicating the degree of slope, than upon the distances between the shading lines. The other method (the French) makes its expression depend more upon the distance between the lines of greatest descent than upon the color produced, although in it also the tint is graduated from dark to light, or white, according to the declivity or level to be shown. These two methods of delineating hills will be considered in the order in which they are named above.

18. When the surface of the ground is illuminated by vertical rays, it is evident that the level or horizontal parts will reflect upwards to the eye (which is supposed to be situated vertically above every point of the map) the greatest quantity of light, for the incident rays then coincide in direction with the reflected ones, the rays of light being supposed to be parallel. But when a vertical ray falls upon a surface inclined 45°, the reflected ray will be horizontal, since both rays make angles of 45° with the line drawn perpendicular to the surface (*Fig.* 8).

19. The natural limits of the declivity of slopes being 45° for the greatest, and a dead level for the least, it is required to apportion the illumination between these limits, taking black to represent a slope of 45°, and white a horizontal plane. Divide the line A B (*Fig.* 9) into ten equal parts, corresponding to the ten steps of a gradation (by 5° at a step), from a level to a slope of 45°. Upon the line C D, below, represent in these divisions, the different inclinations, viz. a level, a 5° slope, a 10°, 15°, &c., to 45°. The level is perfectly white. Then there are nine degrees of color to be determined, from 5°, which is

the lightest shade, to 45°, which is black. Divide now each of the ten spaces of A B into nine equal parts. To exhibit the proportion of black to white in a slope of 5°, make one of those parts black; for 10°, make two black; three for 15°, and sc on to 45°, where all nine parts are black.

20. This scale of color shows the following proportions of black to white for all the inclinations (differing by 5°) from a level to a slope of 45°. All the nine parts of the part of the scale corresponding to the level are white. In a 5° slope, one part of the nine is black, showing a proportion of black to white as 1 is to 8. In 10° two parts out of the nine are black, showing a proportion of 2 : 7, or 1 : 3½. For 15° the proportion is 3 : 6, or 1 : 2. For 20°, 4 : 5, or 1 : 1¼. For 25°, 5 : 4, or 1¼ : 1. For 30°, 6 : 3, or 2 : 1. For 35°, 7 : 2, or 3½ : 1. For 40°, 8 : 1. For 45°, all is black. Expressing these proportions in the form of ratios, we shall have the following table, in which the numerator signifies the quantity of black, and the denominator the quantity of white :—

Level.	No.	*Black.*			
5°	$\frac{1}{8}$	"			
10°	$\frac{2}{7}$	"			
15°	$\frac{3}{6}$	"			
20°	$\frac{4}{5}$	"			
25°	$\frac{5}{4}$	"	or $\frac{1}{4}$ more black than white.		
30°	$\frac{6}{3}$	"	or $\frac{1}{2}$	"	"
35°	$\frac{7}{2}$	"	or $3\frac{1}{2}$	"	"
40°	$\frac{8}{1}$	"	or 8	"	"
45°	all	"			

21. To reduce these ratios to practice in the drawing of shading lines, the four horizontal strips, below the lines E F, G H, I K, and L M (*Fig.* 9), show how the black portion of each set of nine parts is divided up into lines of proper thickness for use. For example, in the strip E G of the rectangle *o r s p* which corresponds to the slope of 5°, the 1 black part to 8 of white, or 1 to 8, is divided into two black lines of half the thickness of the first black part, showing a proportion of 2 : 16. Each of these lines is, in the strip G I, divided into two of half the thickness of those in E G, making four shading lines, showing a proportion of 4 : 32. Dividing these in the same manner

we obtain, in the strip I L, a proportion of 8 : 64, and in the next strip below, of 16 : 128. (See *Par.* 45.)

22. It remains to show the use of this scale, in expressing, by the ratio of black to white, the degree of inclination of a slope. For this purpose the scale must be cut off along the line L M. The part L C D M will then furnish us, along the line L M, with a graduated edge, by means of which the distance between the centres of the shading lines can be marked off, and the line C D will show the different slopes to which the graduation corresponds.

Having now the horizontal sections of a hill given, write upon it with a pencil, in as many places as necessary, the *degree* of the inclination (See *Par.* 10), and bring the line L M of the scale tangent to the upper curve of the zone, at that part of the scale corresponding to the inclination we are required to express, and mark off from its edge the distances between the centres of the shading lines. Through each of the points thus determined draw a *line of greatest descent* (See *Par.* 16). Copy from the scale, for each group of the lines, the exact proportion of black to white, and the color in each zone will then express the degree of the slope, and the line of greatest descent will show its direction.

23. From the foregoing we deduce the following two practical rules. To find the ratio of black to white for any given slope: *Rule* 1. *Subtract the given inclination from* 45° *for a denominator, and take the given inclination for a numerator,* and we shall have the ratio as in the table in *Par.* 20. Apply this, for example, to the expression of a 20° slope. Take 45°—20° = 25 for a denominator. The numerator is 20, hence $\frac{20}{25}$, or $\frac{4}{5}$ is the ratio (see *Par.* 20), of which the numerator represents the black. To find the inclination, the ratio of the black and white being given, or having been observed from a drawing: *Rule* 2. *Multiply the numerator of the given ratio by* 45° *for a new numerator, and add together its numerator and denominator for a new denominator. Reduce the fraction.* For example, in reading a drawing, we find in a certain part a ratio of black and white expressed by the fraction $\frac{5}{4}$. Then

$$\frac{5 \times 45}{5+4} = \frac{225}{9} = 25\text{—the inclination required.}$$

24. In the above calculations, as well as in the scale of shade,

no variations of less than 5° have been regarded; but smaller differences may be used, if desirable, and the scale drawn, and the ratios calculated in the same manner.

25. In representing declivities by this method, considerable practice is required. This should be commenced by drawing repeatedly the scale of shade, particularly the last subdivisions of the black part into lines, so as to apportion accurately the black line to its adjoining white spaces, in such a manner as to express with readiness any of the angles of inclination into which the scale is divided. Then the line may be applied to express the varying inclinations of the same, or of different zones of a hill.

It is not, however, to be supposed possible that every angle can be expressed in the exact proportions of the table of ratios. The most experienced eye and hand are liable to deviate about 2° from the truth, in estimating angles less than 9° and greater than 34°, and about 1° in estimating the intermediate angles. But it is seldom necessary to approach nearer than this to the truth.

26. The other method (the French) of expressing conventionally the degree of inclination, by means of the distances between the centres of the lines of greatest descent, will now be considered. In discussing this system, slopes are expressed by a written fraction, the numerator of which is unity, and generally stands for the vertical distance between the horizontal sections, and the denominator is the horizontal distance between them (or the normal, see *Par.* 10), expressed in terms of the vertical distance as a *unit*. For example, let *a b* (*Fig.* 10) be the profile of a slope, cut by horizontal planes at *a* and *b*; call *b c* 1 (unity), and designate the line *a c* by *s*. Then the slope will be represented by $\frac{1}{s}$. If *b c* is contained three times in *a c*, then the expression for the slope will be $\frac{1}{3}$, and so for any other relation between the base and perpendicular of the right-angled triangle *a b c*. The limits between which slopes are represented in this method, are $\frac{1}{1}$ or 45° for the greatest, and $\frac{1}{64}$, or 0° 53′ 43″ for the least—all slopes less than the latter being regarded as levels. The largest scale that will admit of conveniently drawing the lines of greatest descent, is $\frac{1}{600}$ of the full size, or 6 inches to 100 yards, being about $8\frac{4}{5}$ feet to a mile. In drawings made to this, and smaller scales,

the vertical distance between the horizontal sections is generally taken one yard. For the scale of $\frac{1}{600}$, we shall have, as the least width of zone, $\frac{6}{100}$ of an inch, and for the greatest,

$$\frac{6}{100} \times 64 = 3\frac{84}{100} \text{ inches.}$$

To fill a zone wider, or even as wide as this, with lines of descent, would be inconvenient, and unnecessary.

27. In order to determine the interval between the shading lines for any given inclination (which interval is always reckoned from centre to centre of the lines), we have the following rules :—

Rule 1. *To the distance* (measured along the line of greatest descent) *between the upper and lower curves of any zone, add three-tenths of an inch; a sixteenth part of this sum will be the proper interval for the shading lines.* For example, if the given inclination is $\frac{1}{60}$, the scale being $\frac{1}{600}$, and the zones 1 yard thick, the width of zone for $\frac{1}{60}$ will be $.06 \times 60 = 3.60$, or $3\frac{6}{10}$ inches; to this add $\frac{3}{10}$ of an inch, and divide by 16, and we have

$$\frac{3.60+.3}{16} = \frac{3.9}{16} = 0.244.$$

If the inclination is $\frac{1}{1}$, we have

$$\frac{.06+.3}{16} = \frac{.36}{16} = 0.0225 \text{ inches, \&c.}$$

28. To save the labor of calculation, the following is a practical method of laying off these points by the eye. Cut a rectangle of paper, the side A B, of which (see *Fig.* 11) must be equal to $\frac{3}{10}$ of an inch. It is required to space off the shading lines at MN, which is a line of greatest descent. Apply the rectangle near the middle of MN, with the side AB laid in the direction of the middle of the zone, and towards the left. Prolong AB to the right, and make BE equal to MN. Then AE will be equal to the width MN of the zone, plus $\frac{3}{10}$ of an inch—which distance is to be divided into 16 parts. Now, by the eye (or with compasses, until the eye is sufficiently practised), halve the line AE at *o*, quarter it at *p* and *r*, treat the quarters *Ap*, *po*, *or*, and *rE*, in the same manner, and through each of the 17 points so found, a shading line is to be drawn across the zone.

29. When the curves are nearly parallel, we can easily in this manner, determine seventeen lines at once. But when the curves depart from each other rapidly, the lines of greatest descent also diverge proportionally, and a right line drawn normal to the upper curve, as at G (*Fig.* 11), will have departed from the path of greatest descent GI, when it reaches the lower curve at H. It therefore becomes necessary to correct its direction. This is done by dividing the line of greatest descent into four or more parts, and thus determining three or more medial sections (see *Par.* 12). The true line of slope GI (*Fig.* 11) is thus divided, and by similarly dividing others, as at YZ, &c., we can draw the auxiliary sections *ab*, *cd*, and *ef*. These sections being approximatively parallel, we are now prepared for the application of

30. *Rule* 2. *To a quarter of the distance* (measured as before) *between the upper and lower curves of any zone, add* $\frac{75}{1000}$ *of an inch; a fourth part of the sum will be equal to four intervals.*

Cut a rectangle of paper as before, whose side A′B′ shall be $\frac{75}{1000}$ of an inch. Apply A′B′ to the middle point of the normal drawn at F (*Fig.* 11). Make B′P equal to FK, and divide A′P into four parts, by halving and quartering as before. This will determine 5 lines, to be drawn as far as the auxiliary section *ab*. Apply A′B′ to the left of the second normal KL, and determine four lines of the second auxiliary zone, and so on for LO and OQ.

31. These rules have for example's sake, been now applied to a drawing made on a scale of $\frac{1}{600}$. Let it be supposed that the scale is $\frac{1}{5280}$, or 1 foot to 1 mile. In this scale it is sufficient to consider the curves as being 2 yards apart vertically. Then the least width of zone will be 0.0136 inches nearly, which is very small, while the greatest width will be 0.8704 inches. Now, for a slope of $\frac{1}{1}$ or 45°, the 1st Rule will give for the intervals

$$\frac{.0136+.3}{16}=\frac{.3136}{16}=0.0196,$$

or $\frac{2}{100}$ of an inch, nearly; and for a slope of $\frac{1}{64}$, we shall have

$$\frac{.87+.3}{16}=\frac{.90}{16}=0.05625,$$

or $5\frac{5}{8}$ hundredths of an inch. This is a small scale, and is

adapted to exhibit a portion of the ground from four to eight miles square.

32. Having now established the intervals of the shading lines, it remains to regulate their thickness, or breadth, so as to assist in expressing the declivity. For this purpose, they will vary directly as the inclination, agreeably to the following

Rule 3. *For a slope of* $\frac{1}{1}$, *the thickness of the shading lines is equal to two-thirds of the distance between their centres, and their thickness will diminish with the inclination, down to* $\frac{1}{64}$, *where the lines will be as fine as they can be drawn.* This rule will always, in a slope of $\frac{1}{1}$, make the shading lines *twice the breadth of the white space contained between them* (*Fig.* 12). Thus, for a scale of $\frac{1}{600}$, the zone being 3 feet in height, and the slope $\frac{1}{1}$, we shall have for the intervals

$$\frac{.06+.3}{16}=\frac{.36}{16}$$

two-thirds of which, or

$$\frac{.36\times 2}{16\times 3}=\frac{.72}{48}=.015,$$

or $1\frac{1}{2}$ hundredths of an inch, nearly. If the scale is $\frac{1}{5280}$, the zone being six feet high, we shall have the intervals for a slope of $\frac{1}{1}$,

$$\frac{.0136+.3}{16}=\frac{.3136}{16}$$

two-thirds of which

$$\frac{.3136\times 2}{16\times 3}=\frac{.6272}{48}=.0131,$$

or nearly $1\frac{1}{3}$ hundredths of an inch as before; in both these (and all other) instances, the shading lines decreasing to the finest line, to express the slope of $\frac{1}{64}$.

33. In regard to the length of the shading lines, no absolute rule needs to be laid down. As there is no contemplated relation between the declivity and the length of the shading line, this part of the work is left to the skill and discretion of the draftsman. It may be remarked however, that if we confine their length to the width of the zone, their thickness, in a slope of $\frac{1}{1}$, would sometimes exceed their length; and in a slope of $\frac{1}{64}$, they would often be too long for convenience. In the latter case, this difficulty is obviated by dividing the zone (*Par.* 29); and

in the former, it may be observed, that the lines may be drawn across 4 or 6 zones, more or less, according to the scale, and other circumstances. The extremes of length for ordinary scales, may be set down at $\frac{1}{5}$ or $\frac{1}{6}$ of an inch for the steepest slopes, and about $\frac{3}{4}$ of an inch for the gentlest. Some further practical directions will be given on this subject hereafter.

34. Upon every drawing made in accordance with the above rules, there should be placed a scale, which should conveniently shew the relation between the width of the spaces and the slope of the ground at any part of the map.

The following is a convenient form for this scale:—Divide a line AB (*Fig.* 13) into 64 equal parts, by setting off from A towards B, 64 spaces of (say) $\frac{1}{10}$ of an inch. Number the divisions from A to B, calling the point A zero, and B 64. Let fall perpendiculars from A and B. On that at A, measure downwards to a, a distance equal to the width of four spaces corresponding to a slope of $\frac{1}{1}$, and at B measure downwards to b, a distance equal to four spaces, expressing a slope of $\frac{1}{64}$. Join a and b by a right line. Let fall now, perpendiculars from every point of division on AB, until they intersect the line ab, and the scale is completed.

To make use of it in finding the declivity expressed by the shading lines, take off in the dividers, the width of four spaces at any part of the drawing, and compare it with the lengths of the perpendiculars to A B. If, for example, it is found that it coincides in length with No. 12 of the scale, then the slope expressed by that interval is $\frac{1}{12}$.

35. Reciprocally, this scale may be used to determine the width of four spaces. For, having constructed it, as before, in the same scale of distances as the map to be drawn, and knowing as we do, the expression for the slope at any part of the drawing, the perpendicular of this scale of intervals, which agrees by its numeration on A B with the denominator of the slope-ratio, will evidently be the corresponding width for four spaces at that place.

36. A still more convenient practical method of marking off the intervals, and drawing the shading lines of the proper thickness, consists in the use of a tangent movable scale, to be applied along the horizontal curves. It shows the relation between the length, thickness, and the intervals of the shading

lines for all slopes. Its construction and use are as follows:—Draw a right line A B (*Fig.* 14). At A, draw perpendicular to it, and downwards, a right line, whose thickness and length shall both correspond to a slope of $\frac{1}{1}$. This line must be drawn according to the scale of the map. For example, if the scale of the map is $\frac{1}{1200}$, or 1 foot to 400 yards, the horizontal curves being 3 feet apart vertically, or $\frac{3}{100}$ of an inch, according to the scale; then (*Par.* 26) the width of zone for a slope of $\frac{1}{1}$, is .03 inches. The length of the shading line will then be .03, and its thickness (by Rule 3) is .0137 inches. Having drawn this line, of the proper length and thickness, lay off towards B, the first interval of the scale, which must correspond to a slope of $\frac{1}{1}$. By Rule 1 (*Par.* 27), we have

$$\frac{.03+.3}{16}=\frac{.33}{16}=.0206$$

for this interval, and for $\frac{1}{64}$ we have .1387. Now, from $\frac{1}{1}$, the slope expressed by the former distance, the intervals go on increasing uniformly up to $\frac{1}{64}$, which is expressed by .1387. They may, therefore, be regarded as an arithmetical series, whose first and last terms are .0206 and .1387, respectively, and whose common difference is

$$\frac{.1387-.0206}{63}=.001875.$$

By adding this common difference 64 times to the first interval (.0206), we shall, by laying them off from A, obtain the successive intervals of the scale. When the scale is very small, the common difference may be doubled, and the intervals laid off from A, by two at a time, and afterwards halved; or it may be quadrupled, and four spaces laid off at once, and then divided into four equal parts. Under such circumstances, these will approximate, very nearly, the true division.

This, by successive additions, is the most accurate method of setting off small distances. Let fall, now, through these points thus determined on the line AB, perpendiculars to AB, of indefinite length, and varying uniformly in thickness, from the first line at A, which is .0137 of an inch thick, to the last one at B, which is as fine as possible. To determine the lengths of these shading lines, or in other words, the width of the zone to which their intervals and thickness correspond,

start from the first line, already drawn of the proper length ($\frac{3}{100}$ in.) at A, and from that line increase the length of each one, up to the 64th, in an arithmetical progression, whose first term is .03 the least width of zone, and the last 1.92 the greatest width. The common difference of this series is .03, and by successive additions (as in spacing off A B), the increasing lengths of the perpendiculars may be laid off. Having done this, draw through their lower extremities the curve D C, and the tangent scale will be completed. Each division of the line A B is the interval between two shading lines, corresponding to a width of zone equal to the mean of the two perpendiculars which include it.

37. To make use of this scale it must be cut out, following its outline, A B C D A. Place the line A B on the drawing, tangent to one of the horizontal curves, so that the perpendiculars to A B shall be normal to the curve, and coincide in their length with the width of the zone. Let the lower curve of the zone cut the curved outline C D, midway between two of the perpendiculars. See *Fig.* 14, at *m n* and *o p*, where the adjustment of the scale at *m n* gives the interval and thickness of the shading lines by means of the perpendiculars, *m o* and *n p*, which are in length and direction the normals of the zone *e f*, *g h*, at *m* and *n*. This scale may thus be applied to determine groups of lines in different parts of the hill.

38. Although the representation of the ground is thus conformed to geometrical rules at all points, it must not be thought necessary to repeat the process of construction for every line. It will suffice to do so at those places where the slope exhibits the greatest variations. Thus, a group in each zone will be constructed where the slope is least, and again where it is greatest, then a few intermediate ones. By graduating the changes in the shading lines, in passing from group to group, both as to their thickness and their intervals, we can easily fill the vacancies between the determinate groups. Without being mathematically exact, we shall thus obtain a result sufficient for all practical purposes, and as accurate as can reasonably be looked for in employing the lines of greatest descent.

39. Tabular view of the width of zone, the intervals and thickness of the shading lines, corresponding to different scales, and different heights of zone:—

Scale.	Height of zone.	Least width of zone according to the scale.	Greatest width of zone.	Greatest interval of shading lines.	Least interval of shad'g lines.	Thickness of lines for 45°.	
$\frac{1}{600}$	1 foot.	.02	1.28	.09875	.02	.01333	1 in. to 50 ft.
$\frac{1}{600}$	3 feet.	.06	3.84	.25875	.0225	.015	" "
$\frac{1}{1200}$	3 feet.	.03	1.92	.13875	.0206	.0137	1 in. to 100 ft.
$\frac{1}{1200}$	6 feet.	.06	3.84	.25875	.0225	.015	" "
$\frac{1}{5280}$	6 feet.	.0136	.8723	.07327	.0196	.0131	12 in. to 1 mile
$\frac{1}{15840}$	11 feet.	.0083	.5212	.0513	.0192	.0128	4 in. to 1 mile
$\frac{1}{15840}$	22 feet.	.0166	1.062	.0851	.0198	.0132	" "
$\frac{1}{80000}$	60 feet.	.00075	.048	.02175	.0188	.0125	n'ly 8 in. to 1 m.

To calculate the values of the above different quantities, first find the height of zone expressed in parts of an inch, according to the scale of the drawing. For example, suppose the scale to be $\frac{1}{1200}$, and the horizontal curves to be 3 feet apart vertically, that is, that the height of zone is 3 feet. Since 1200 feet on the ground is represented by 1 foot of the drawing, we can write 1200 ft. = 12 in., or 100 ft. = 1 in., or 100 yds. = 3 in., whence 1 yd. = $\frac{3}{100}$ inches, or .03 is the height of the zone according to the terms of the scale. Then the least width of zone (that for a slope of $\frac{1}{1}$) will be .03 inches. The greatest width (or $\frac{1}{64}$) will be $.03 \times 64$, or 1.92 inches. By Rule 1 the greatest interval will be

$$\frac{1.92+.3}{16}, \text{ or, } \frac{2.22}{16}=.13875$$

The least interval will be

$$\frac{.03+.3}{16}, \text{ or, } \frac{.33}{16}$$

which is .0206. And by Rule 3 the greatest thickness of the shading line will be $\frac{2}{3}$ of .0206, or .0137. It will be observed that the actual height of zone is varied according to the scale of the map. This is necessary, in order to give room to express the curves of a steep slope on a small scale; and if the height of zone and the scale bear a proper relation to each other, the

intervals and thickness of the shading lines for a slope of 45° will be nearly the same for all medium scales, only the maximum intervals changing as the scale is enlarged or diminished. (See *Figs.* 17 and 18 for examples on scales of $\frac{1}{1200}$ and $\frac{1}{5280}$ respectively. The vertical distance of the curves is three feet in *Fig.* 17, and six feet in *Fig.* 18.)

40. Although in both of the methods of treating lines of greatest descent, just explained, the *theory* of the convention is perfect, yet it is admitted that in practice an approximation only is obtained, inasmuch as a great deal is left to the judgment of the draftsman's eye, and his skill of hand. Still, it is not only useful, but necessary, to have some fixed principles as a basis on which the art may be founded, and to which practice may be conformed. The student ought, therefore, to make himself perfectly acquainted with the method (whichever it be) that he adopts, so as to present throughout his drawings a consistency in the expression of at least the *relative* degrees of inclination. For popular use, on ordinary occasions, this will be sufficient, as almost every one who is interested in looking at a topographical map, will have learned enough of the conventional signs to comprehend their general intention; while the scale of shade, or of spaces, which should always be put upon a drawing, will inform those who are disposed for a minuter reading of it, if it is geometrically correct.

41. But as it is not always possible, either for want of time, or for other reasons, to attempt a complete realization of the theory of these methods, it is proposed to exhibit their different modifications between strict conformity to the rules above stated, and those for rough or rapid sketching of the ground from nature.

42. With regard to the English method, or that of vertical illumination, it has been remarked, in applying it in the service of the U. S. Coast Survey, that "this scale of shade does not represent slopes greater than 45°, thereby limiting the graphic capabilities and effect of the map. It also makes the slopes too dark as they approach the inclination of 45°, and does not well represent slopes of less than 5°, which latter it is often necessary and desirable to express distinctly." A scale is then to be sought for by which the lower slopes may be readily distinguished from one another, and the graphic effect of the

high slopes retained. The slopes $\frac{1}{26}$, $\frac{1}{10}$, and $\frac{1}{5}$, or from $2\frac{1}{4}°$ to 11° (about), are most frequently met with in those parts of our country not mountainous. Slopes from $\frac{1}{40}$ to $\frac{1}{10}$, or from $1\frac{1}{4}°$ to 6°, are such as are usually surmounted by roads. From a level to 25°, it is necessary to represent more slopes, and to represent them more distinctly than between 25° and 45°, and between the latter limits it is not necessary to distinguish nicely between every few degrees of slope. Between 45° and 90°, slopes should be represented for graphic effect, such as for clay banks, rocky precipices, &c., but it is not necessary to show their variation for any other purpose.

43. The scale then proposed, adapted to any part of our country, both for civil and military purposes, and as affording the means of graphic representations of all slopes useful for such objects, is the following modification of the scale of *Par.* 20: —

Slope.	*Proportion of*	
	Black.	*White.*
$2\frac{1}{2}°$ or $2\frac{3}{4}°$	1	10
5° or 6°	2	9
10° or 11°	3	8
15° or 16°	4	7
25° or 26°	5	6
35°	6	5
45°	7	4
60°	8	3
75°	9	2

A slope of $1\frac{1}{4}°$ is thus represented by the same thickness of shading lines as that of $2\frac{1}{2}°$, but the intervals should be doubled in the latter case, and so proportionally for any slopes less than $2\frac{1}{2}°$. By this scale the slighter slopes are represented well, and will be readily distinguished from each other. The shades for slopes less than 16° are darker than in the scale of par. 20, which renders their differences more observable. From 25° to 75° the shades are lighter than the corresponding ones of the other scale, the distinction being here of less importance. The white space, for a slope of 75°, does not in this scale become too small for practical use. A scale in accordance with the proportions in the above table, may be constructed as is described in *Par.* 19, except that there must be eleven instead of nine divisions on A B, used for proportioning the black to the white. Its practical application is effected as in *Pars.* 21 and 22, by repeated subdivisions of the proportion of black, until a proper thickness of shading line is arrived at.

44. A further modification of the scale of shade has been very generally adopted in England, which for ordinary purposes has the advantage of simplicity, and facility of applica-

tion. It consists in establishing with accuracy only three proportional quantities of black and white, for three medium slopes, viz. 15°, 22½°, and 30°, a level being represented by white, and a slope of 45° by bringing the shading lines in contact, or perfect black. Thus by Rule 2d, *Par.* 23, we have this table of slopes with their proportions of illumination :—

Slope.	*Proportion of*	
	Black.	*White.*
level.	—	all.
15°	1	2
22½°	1	1
30°	2	1
45°	all.	—

A scale of shade may be at once constructed from this table, by assuming the thickness of the shading line for the medium slope of 22½°, which thickness must be suitable to the scale, and to the degree of fineness or finish, which it is intended to give the drawing. Having drawn this portion of the scale (as at A, *Fig.* 19) with equal proportions of black and white, diminish by one third the thickness of the black line for the part B of the scale, which will then correspond to a slope of 15°; and increase its thickness by one third for the portion C of the scale, which will represent a slope of 30°, white and black being respectively the extremes. This scale should be carefully applied to the map wherever the slope to be represented corresponds with one of the three in the scale. All intermediate inclinations are of course indicated by graduating the thickness of the shading line, referring it to the regulators in the scale. (See *Fig.* 35 for a drawing made according to the vertical system.)

45. In the English and German method, and in all its modifications, the thickness of the shading line in the medium slopes (that is, where it is nearly equal to the interval between the lines), is an arbitrary quantity, and is regulated by the scale of the drawing, by the skill of the draftsman in mastering bold or fine lines, by the time that may be spent in drawing the many or the few shading lines required, &c., &c. Generally, if the lines have such a relation to the scale of the drawing as to present a well connected appearance, it will be found that fewer shading lines, and a rather coarse texture will conduce more to clearness of expression when viewed at ordinary distances, than a finer texture, which has a tendency to dryness of style. In the French method, however, as has been before remarked, the thickness of the shading lines and their intervals are both fixed by the scale of the drawing.

46. The scale of shade may evidently be still further simplified, or, as in sketching in the field, it may be reduced to a mere graduation by the eye, of the color for the different slopes that offer themselves for description. As much of the data for topographical drawing, relating to extended civil works, is collected by sketching in the field, a few practical remarks on that subject will now be offered.

47. Field sketches are made with the lead pencil, and may be drawn upon every page of the compass-book, or upon the alternate pages, at the option of the topographer. In the former case, the bearings and distances are recorded upon the drawing (*Fig.* 16½); in the latter, the record occupies the left hand page, and the sketch the opposite one. The page for sketching should be ruled in squares, with blue or red ink, forming thus an indeterminate scale, the length of the sides of the squares being assumed at pleasure, according to the nature of the ground. Both the record and the sketch are read from the bottom of the page upward. Suppose the stations of the survey to be one hundred feet apart; then, assuming the side of the square to be one hundred feet, commence the sketch at the bottom of the page—in the centre, if the survey promises to be tolerably straight; if otherwise, at some point to the right or left of the centre, the reason for which will be explained directly. Let the bearing from the first station (the starting point or zero) be N. 10° E. (*Fig.* 16½.) Draw a line from the bottom of the page upwards; the side of the square being assumed one hundred feet, number the stations upon the squares as far as the line is run, say three hundred and twenty-five feet, and write the compass angle down along this line. Let the bearing from the second station, or No. 1, be N. 1° W.; draw a line making, as nearly as can be judged by the eye, the proper angle with the last bearing, and proceed as before. When the page is exhausted, commence with a vertical line at the bottom of the next one, marking upon it the remainder of the old bearing, and making, by the eye, a new series of approximate protractions as before. If it can be foreseen, as in most cases it can, that the line of survey will be very crooked, bending, for example, from left to right, then commence the bearing at the bottom of the page accordingly, beginning at a point on the extreme right (*Fig.* 16½, dotted line), and running it dia-

gonally to the left, so as to make due allowance for the great deflection anticipated in the next bearing. Such cases may be foreseen in running around an inclosure, or in following a curving stream or ridge. The advantages of the system of squares in sketch-books completely overbalance the one disadvantage which is, that the diagonal bearings will not make exact distances upon the squares, while the vertical and horizontal ones will. It will be remembered that the surveying book is designed to be exact only in its *record* and the general features of the ground, and that a slight change of scale is not material, as it can be made exact when the survey is protracted upon the map. By these approximate protractions, any page of the book of survey conveys a very just notion of the bearings and distances, and of the relative positions of the general features of the ground. The first station being at the bottom of the page, (*Fig.* 16½,) note down in the space between it and the second one, all the features of the ground passed over by the line of survey; as to whether it is cultivated, forest, marsh, &c.; whether it is crossed by streams, ditches, &c., and their width; if it rises or falls, about what degree of slope, &c. On both sides of the line introduce, according to the scale, and their distances, as judged by the eye, all topographical objects within sight, such as buildings, roads, streams, hills, &c., &c., drawing them to the scale if possible, and if they cannot be got upon the page, describing briefly their nature and position. In sketching hills endeavor to project as many horizontal curves as possible, which should be lightly put in, and then the shading lines may be drawn over them. The degree of slope should be frequently written down in numbers upon the sketch. The names of localities, streams, hills, farms, &c., should also be entered.

48. Thus far we have supposed a measured line upon the ground, to which the situation and dimensions of objects might be referred. It is much more difficult to embody the relative positions and dimensions, where all is left to the eye. Here a cultivated judgment is of the greatest value. Practice alone can make a good sketcher under such circumstances. Rules must, from the nature of the case, be few and general. In the first place, all objects within the field of vision are presented to the eye in *perspective*, whereas the sketch is to be a *plan*.

The apparent diminution of dimensions in distant objects must therefore be corrected on the plan. For example, the windings of a crooked stream, or a road, in perspective, are much exaggerated in retiring into the distance; they must therefore be *straightened out* in the sketch, more and more, as they are more removed. 2nd. In looking at variously placed hills from a somewhat elevated station, the eye will in some cases look directly, or perpendicularly, at the face of some slopes, while, in others, the surface of the slope, if prolonged, will pass through the eye, and will not be seen in its true dimensions, though its inclination may be judged. In sketching the shapes of hills, bodies of water, masses of forest, &c., these facts must be taken into consideration, and to insure skill, eye-sketches of a small portion of ground having well-marked features, must be frequently made, and compared with measurements of the same features. In sketching a single hill, the best station is at the summit. First endeavour to represent the lowest horizontal curve of its surface; then a medial one; then the form of the level space at the summit, or the highest horizontal curve. Others may then be introduced between these, until the ground is sufficiently expressed. The angles of inclination should be frequently noted down in numbers: all accidents of ground, such as ravines, rocks, &c., should be carefully placed, and all other objects, such as houses, fences, trees, &c., should be put down in their proper relative positions and dimensions. Having thus prepared a skeleton of horizontal curves, numbered as to inclination and heights, the sketch will always serve a useful purpose without any lines of greatest descent. After sufficient practice in this method, the eye will become so cultivated as to enable the draftsman to express the form of ground by lines of descent at once, the mind conceiving the position of the horizontal curves, and thus supplying the necessary data for the shading lines, the relative thickness and length of which for the different slopes, is a matter very easy of acquirement. But this should not be attempted until the method by horizontal sections is thoroughly mastered.

49. It is easy thus to make a correct sketch of a single hill, but when there are many, and the general face of the country is sloping also, the difficulties of representing the connexion of the different hills at their bases are considerable. In such cases,

the direction and lengths of the valleys (or water-courses if there are any) must first be noted, bearing in mind the illusions of perspective in both its effects, mentioned in par. 48. Then establish the positions of the different summits, marking down their relative heights, after which put in the other objects to be represented, such as roads, trees, buildings, &c., &c., referring their positions to each other, and correcting them where they are found to disagree. Horizontal curves present the readiest means to the beginner in sketching declivities. When, after some practice, the form of a body suggests (as it always will) its horizontal sections (see *Par.* 11), then it will be time to resort at once to the lines of greatest descent. The greatest difficulties to be overcome in the practice of eye-sketching are, 1st, that of converting a perspective view into a *plan*, in all its true proportions; and 2nd, in forming a just conception of the intersections of different slopes *at their bases.* Hence the rule, to project first upon the sketch, all the lowest lines, or water-courses, and then the highest parts or summits. Then the middle lines and objects may be placed, and the sketch filled up by referring all others to those three groups which may be regarded as determined.

50. The lead-pencil for field drawing should be moderately hard, and the general tone of the drawing should be rather light. The shading of slopes ought not to overpower by its depth the distinctness of other objects, and the pencil should be so used and of such a quality, as not to be easily defaced by rubbing.

51. In concluding these remarks upon the methods of representing hills by the conventional use of lines, drawn with pen or pencil, it is necessary to refer to the mode of expressing inclinations that are steeper than the "natural slope," or greater than 45°. Such slopes frequently occur in clay banks, steep ravines, and in rocks. As they are always exceptions to the law of slopes, and save in the case of rocks, cannot be regarded as a permanent form of the ground, since they are constantly undergoing reduction by the action of natural causes, the method generally adopted to show them, is to make their shading lines exceptional also. In the Horizontal System this is done in earth slopes, by shading them with a set of black and short lines drawn perpendicular to the horizontal sections;

that is, in a direction contrary to the general motion of the shading lines of the drawing. They should be made as black as possible. (*Fig.* 7, at B.) For the same kind of slopes in the Vertical System, their shading lines, also as black as possible, are drawn parallel to the horizontal curves, or contrary to the general tenor of the shading lines of the drawing. (*Fig.* 15, at A.) These methods answer also for the expression of occasional rocks, particularly projecting horizontal strata; but when the slope is entirely rocky, even if it be not steeper than 45°, the shading lines are thrown in, in every possible direction, not intersecting, but abutting abruptly upon each other, in short heavy masses. (*Fig.* 15, at B.)

52. The following are some practical remarks and directions for shading hills according to the Vertical System. All the preparatory pencil lines having been drawn lightly, and the spaces for the shading lines being laid off by dots, begin to shade at the steepest part of the upper zone of the hill. Draw the shading lines firmly, from curve to curve (introducing auxiliary curves for that purpose wherever it is necessary), so that each row of lines may terminate evenly at the lower edge. Draw always toward the body, turning the drawing on the table for that purpose. Draw these shading lines always *down the slope*, and proceed with them from left to right, so that the line just drawn may be uncovered by the pen, and the distance to the next one be seen. Go all around the upper zone in this way, finishing by joining the row at the point of setting out. This is always effected more easily and neatly in the steepest part of the slope. After finishing the first zone, proceed to the second, and so on to the foot of the slope. Where the curves are nearly parallel, the shading lines are straight, and also nearly parallel; but when the curves depart much from each other, the shading lines, being lines of greatest descent, must be normal to the curves, and will therefore themselves have some curvature, in order that they may tend perpendicularly upon a second curve, which is not parallel to the first. (*Fig.* 15, at C.) In such cases it is necessary, besides drawing the medial auxiliary curves, to put in lightly with a pencil, at short distances, a number of normals (*Fig.* 15), which, being carefully studied, will tend to correct or confirm the directions of the shading lines as they are drawn. In introducing additional curves in those parts

of the hill where the slope is slight, care must be taken not to shorten too much the length of the shading line, for where the interval between them is large, the line must be proportionately finer and *longer*. (See *Par.* 33.) Any change in the direction, thickness, or proximity of the shading lines, required by the different inclinations, must be effected gradually, and all sudden changes of that nature must be carefully avoided. Any two consecutive lines of any part of the hill, should be sensibly equal, similar, and parallel. They should be twin lines. The same thing is to be observed of contiguous zones; any change in their color or expression, must be made gradually. If it be required to pass from a light zone to a dark one below it, make the lower extremities of the lines of the light zone a little heavier, so that they may meet the upper extremities of the lower row, with nearly the same color. The latter may also be lightened a little. As zones differ in inclination, so of course will the spaces between the shading lines. No attempt therefore ought to be made to prolong the lines of one zone into the zone below. The lines in each row are manifestly independent on each other in that respect. It is only necessary, as above stated, to avoid sudden changes of colour in passing from zone to zone. Even in a perfectly uniform slope, it will not do to prolong the lines thus, because it gives a hard and bad character to the style. But in the case of a conical hill, as in *Fig.* 16, it would give rise to an error in principle; for we should, soon after leaving the summit, have too few lines of descent to cover the ground, and they would soon be so far separated as to lose their connexion, degenerating into a great meagreness of style. The same rules, in joining the different rows of lines, are to be observed here, as in the Horizontal System, viz. the extremities of one set of lines must not protrude within a neighbouring set (*Fig.* 20), nor must a vacancy be left (*Fig.* 21). The rows must be accurately joined, without showing either a white line at their junction, as in the latter case, or a dark one, as in the former (see *Fig.* 22). The method of joining them is shown in *Fig.* 23 on a large scale, where the lines of the lower row, coming between those of the upper, start from a line which connects the lower extremities of the upper row. When the whole plan of the hill has thus been covered with lines of greatest descent, the base and summit must be softened off, by

tapering to a fine point the lower end of each line, at the base, and doing the same at the summit, by turning the drawing upside down, and tapering the upper end of each line of the upper zone (see *Fig.* 15). The whole hill being finished, the pencilled lines may be removed. The same directions here given for executing the drawing of a hill, will apply to a hol low, the shading lines of which are converging.

53. We will now proceed to consider the conventional use of lines in the representation of other features than hills, which alone have engaged our attention thus far; and 1st, of *water*. When the horizontal sections of the ground are continued, by means of soundings, below low water-mark, that part of the drawing covered by water must be left entirely white, so as to allow a clear exhibition of the lines of soundings, and the horizontal curves of the bottom, as determined by their means. But when the delineation of the ground extends no further than the water surface, then for the sea, large lakes and rivers, the method of shading by lines drawn parallel to the shores, and graduated outwards from them, is generally used in the vertical system. This mode of proceeding would not answer in the horizontal system, as there would then be no distinction between land and water. For the latter system, the method has been explained in par. 15. In order to execute symmetrically this style of shading water, the following directions must be strictly observed. After having drawn, with uniform thickness, a moderately stout line, as at A, *Fig.* 24, for the outline of the water, throughout the whole drawing, begin by applying to it, as closely as possible, the first shading line. In order to do this, attend to the narrow white space between the two, making it a *fine white line*, and of *even width*. The first shading line may be nearly of the thickness of the shore line, and should follow it closely in its minutest deviations. Apply such a line to *every shore line* in the drawing. When that is done, proceed to the second shading line, which may be a little finer than the first, and a very little more removed from it than the first was from the shore-line. Carry this one throughout the drawing, in the same manner as the first one. Then take up the third line, increasing, by very little, the distance between the lines, and drawing it a little finer. In this way, go on, adding one line at a time to every shore on the drawing, in-

creasing very gradually their intervals, and diminishing, as gradually, their thickness. These lines should be drawn *clean*, and as steadily as the hand can make them. Take a very short hold of the pen, resting the point of the middle finger upon the paper. Each line should be of uniform thickness throughout its length, and kept constantly at the same distance from the one last drawn. Draw always towards the body, turning the drawing as before directed, and keep always the line last drawn *to the left* of the one in progress, so that the distance between it and the point of the pen may be constantly watched. The lines must be *completed* successively, as above directed, for the following reasons;—1st. The eye becomes accustomed to the interval employed, and thus the confusion attendant upon carrying on at the same time, three or four lines having different intervals, is avoided. 2d. By this equal distribution of the lines, symmetry is insured, because whatever be the width of different channels (*Fig.* 24, at B), an equal graduation of tint from every shore is obtained, and the shading lines meet in the middle, which might not always be the case otherwise. 3d. It enables us to conform to a principle, which is, that every line must *return to itself*, and no *spiral* lines are admitted. But sometimes two lines will coalesce, as at *c c*, *Fig.* 24, where they join into one at *d*, and afterwards separate into *e e*. The last line, in the middle of a piece of water, must be a line returning to itself, and not a spiral. When the shading lines meet the margin of the drawing, they are cut off; but they are drawn as if they were to be continued out of the margin. These instructions may seem over minute, but the beginner must be warned that this is the most tedious and uninteresting part of topographical drawing, and requires great care and patience, in order to produce a good effect. High and low tides are represented thus (*Fig.* 24). bis. Draw the shore line for the high water-mark, and the shore line for the low water-mark. Then commence from the high water line, graduating outwards until the low water line is met, which must be regarded as a *margin* for the shading lines, and in which they must terminate. Then commence anew from the low water-mark, with a new graduation, which is carried on uninterruptedly.

54. Smaller bodies of water, such as ponds or pools, are

represented by a tint of fine, unbroken lines *ruled* across them parallel to the base of the drawing (*Fig.* 25). Rivulets, or very small streams are represented by one, two, or three lines, according to their magnitude (*Fig.* 25, at A, B, and C).

55. The sign for marsh is composed of a mixture of the two signs for water and grass land, as in *Fig.* 26, where the water is indicated by fine and full lines, ruled always parallel to the base of the drawing, and grouped in an irregular manner, so as to leave small islands interspersed through it. These islands are filled with grass, drawn as will hereafter be described, with here and there a tree. The division between the land and water should be first sketched lightly with a pencil, as a guide for ruling the lines. The texture, as to fineness, of this sign should be regulated by the scale of the map, and be consistent with the other lines in it, in that respect. Distinctions have sometimes been made between salt and fresh marsh, deep morass, &c.; but this ruled sign is the neatest form, and does not charge the memory with nice distinctions, which can easily, if required, be expressed in small lettering, upon the drawing.

56. *Forest*, being one of those features whose conventional sign is intended to suggest, in some degree, the object itself, is represented by characteristic lines, resembling a pen drawing of a tree as seen in horizontal projection, with its shadow upon the ground, cast by parallel rays inclined 45° to the horizon. The only distinction between the various forests, that can be recommended, is to reserve a particular sign for *pines*, and to include all other kinds in one character of foliage. *Fig.* 27, at A, shows a pine forest, where the sign has a star-like form, and is darkened towards the right hand and lower side, where the shadow is placed. At B is the sign for all other forests, where the character of the outline is round, with a few touches of the pen on the interior, and towards the shadow. This outline, to have a good effect, should be made with simple curves, firmly drawn, and not be cut up by smaller indentations, as in fig. 27 (bis), the bad forms of which are also to be avoided. The trees, and masses of trees, must be disposed in every possible variety of position, avoiding carefully all arrangement in lines, or regular figures. Their size depends upon the scale of the map, and should consist with the dimensions of buildings, the width of roads, &c. The sign in *Figs.* 27 and 7, at C, indicates an

ornamental grove, shaped by art, and is used in representing parks, gardens, &c. *Orchards* are shown by placing single trees, with their shadows, upon the intersections of two sets of equidistant parallel lines, drawn at right angles with each other, as in *Fig.* 28. One set of these lines is usually placed in the direction of one side of the inclosure. The lines should be drawn in pencil, and afterwards erased. Single trees are drawn, as shown in *Fig.* 29. The shadow is detached from the outline of the tree, and is intended to have an elliptical form. When the scale is small, a single tree becomes a simple circle, touched with the pen, on the side towards the shadow. Some topographers prefer to draw trees in elevation, as in fig. 30. This method certainly renders it easier to describe varieties of foliage, and may be used with as good effect as the other, according to the fancy of the draftsman. When trees occur upon a hill-side, the shading lines of the hill-side should be interrupted, in order to receive the body of the tree, but not its shadow, which may be drawn independently of them when the slope is slight; but when it is steep, the shadows may be omitted, and the trees must be shaded dark, so as to be nearly of the same color as the shading of the slope, and, at the same time, the forest may be represented as less dense than it is usually drawn upon a more moderate inclination.

57. *Cleared land, grass, or meadow*, is indicated by groups of short lines, arranged like tufts of herbage. Each of these little figures is drawn with its base *always parallel* to the base of the drawing, whatever may be the shape of the inclosure containing this sign. They must likewise be spread evenly over the paper, not too thickly, and all appearance of regularity or approach to arrangement in lines must be avoided, as directed for forest (see *Fig.* 31). Each tuft is composed of from 5 to 7 lines, drawn as if converging in a point below the base, as is shown on a large scale at A—the middle lines being the longest, and the extremes mere dots. The base must be a straight line, not curved, as at B.

58. *Cultivated land* is represented by alternate broken and dotted lines, suggesting furrows. These are, for the sake of variety, made to assume different directions, one set of them being generally parallel to one of the sides of the inclosure (*Fig.* 32). The ground must first be prepared by drawing light pencil lines

equidistant from, and parallel to, each other, in sets. These pencil lines must be ruled, and if they cannot be spaced off by the eye, their intervals must be determined, and marked by dots. Then draw the broken and dotted lines *by hand*, over the pencilled guide lines. The joints in the broken lines must not be opposite to each other, and the *breaks*, or vacancies in the lines, must be very short. The dots of the dotted lines must be made by touching the point of the pen to the paper, and immediately lifting it off, without *dragging* it over the paper; this will make a round dot. The dots must be fine, and close together. As before remarked, it is scarcely necessary to distinguish between the different kinds of cultivation, as it is the most variable of all topographical features. If it be desirable to describe the existing crop, it can always be done by a few words, neatly lettered on the drawing.

59. *Uncultivated land*, which is not forest, but such as brushwood, heath, prairie, &c., is represented by mixing tufts of grass with small bushes, of less dimensions than those of the trees in the sign for forest (*Fig.* 33).

60. *Sand and gravel* are shown by dots—the latter rather coarser than the former. In making these, the pen should not be jerked over the paper at random, but slowly put down and taken up, and never without an *intention* in regard to the position of every dot. Make them as directed in cultivated land, and avoid all arrangements in lines (*Fig.* 7, at D). In sand-hills, let the sides of the slopes be made darker than the level parts, by making the dots closer together, in order tc produce a deeper tint, or shade.

61. The different forms of signs for buildings, inclosures, roads, &c., &c., are given in *Fig.* 34. The equidistant lines representing a road, can, when curved, and therefore to be drawn by hand, be conveniently drawn thus:—Take a right-line pen, and open its two blades to the desired width of the road; then place some India ink upon the inner face of each blade, in such quantity as not to allow the opposite masses of ink to run together. Then, if the pen be drawn over the paper, the two points will describe two equidistant lines.

62. It must be remembered that the dimensions of all these signs are variable, and must correspond with the scale of the drawing. This just proportion may be gained in considering

the projected dimensions of buildings, their class, as to being farm houses, mansions, public institutions, &c.; for it woul not be consistent to make a tuft of grass as large as a country seat or a college.

63. All that has been said in regard to the manner of executing the conventional signs with a pen, will apply equally to sketches, or even finished drawings, made with a pencil or any instrument which produces lines. But there is another method of topography, now coming into very general favor, viz. that of expressing the different states or conditions of ground, and even its variations in level, by means of *colors*, the manner of using which will now be considered.

64. The first thing necessary to be done, in order to prepare a sheet of paper for a tinted drawing, is to strain, or stretch it, so that it will not remain blistered after being wetted by the laying on of the tints. For this purpose the paper must be moistened, and laid flat upon a smooth and clean board, and before it dries, the edges must be fastened down with glue, or very stiff paste. By moistening the paper, it is expanded, and if its edges are secured while it is thus enlarged, its shrinking in the act of drying causes the strain, which keeps it flat, and enables it to restore itself in drying, after it has been blistered by wetting.

65. Having thus prepared the paper, the lines of the drawing will be put upon it, first in pencil, and afterwards with a *very fine* ink line. The ink, although black, must not be thick; for, when the lines (outlines only) of all the shores, roads, buildings, &c., are drawn, and all the pencilled lines erased, then the drawing must be *washed*, either by exposing it to water dashed over it, or by quickly passing a soft sponge, well saturated, across its surface. This is done for the purpose of removing those portions of the ink, which a wet brush would detach from the paper in laying on the colors, and which, mixing with the tint, would injure its purity. When dried, the drawing will be ready to receive the conventional tints, which are expressed by the following colors, either used singly or compounded.

66. They are five in number, viz. *Indigo*, *Carmine* (or *Crimson Lake*), *Gamboge*, *Burnt Sienna*, and *Yellow Ochre.* In using and mixing these colors, it must be observed that Indigo is the

most powerful, and Gamboge the weakest; and that Yellow Ochre does not combine well with any of the others, but is used alone.

67. Before proceeding to consider the significance of these colors and their combinations, it is necessary to explain the mechanical conditions to be fulfilled, and the rules to be observed, in order to insure neatness and facility of execution. When a tint is to be mixed, let the end of the cake of color be wetted, and allowed to soften for a minute or two. This will cause it to rub smooth, and free from small fragments. Then *moisten* (only) a perfectly clean plate or saucer, and rub a sufficient quantity of the color upon it, as much as will tinge to the proper intensity (say) three or four tablespoonfuls of water, which being added to the color, and mixed by the brush, the tint is ready for use. Let the paper be inclined about five degrees to the horizon, the lower edge towards the body. This is done that the color may flow easily over its surface, for the whole art of laying on a *flat tint* consists simply in allowing the colored water to flow over the paper, which is uniformly tinged as it passes over. To spread the color, begin with a full brush at the top of the figure (suppose it to be a rectangle), and cause it to lie neatly along the upper outline, then strike the brush from left to right, and from right to left, alternately, bringing the tint down in horizontal bands or stripes, controlling it neatly and exactly within its proper outlines, and keeping the surface of the paper well wetted with the tint. As long as this is done, the tint can be carried on with perfect continuity. On arriving near the lower outline of the figure, the quantity of tint must be diminished, so as to leave just enough in the brush to finish, without allowing the color to accumulate upon the lower outline. In no case, anywhere on the drawing, must the color be allowed to lie in puddles, or drops. The art of laying a flat tint is so easily acquired, with a very little practice, that the only difficulty in it is the outline, to conform to which requires some care in using the point of the brush. When the colored water has once thus flowed over the paper, the tint is *finished*, and must not be touched again; for if there be any little defect in it, one trial will show that any attempt to remedy it while the color is drying, will only make it worse. In laying tints of complicated shapes, the

operation is much facilitated by first moistening the paper, and working upon it just before it dries. As the hand acquires skill, it will be found that, generally, tints are better in proportion as they are more quickly laid on, and with less of color in the brush. Should stains or patches occur, however, they may be remedied by wetting the whole drawing, and gently washing the faulty parts with a brush or soft sponge, and repeating the tint lightly, should it be too much reduced by washing. Tints that are too strong may be rendered weaker by the sponge, in the same manner, and some may even be removed entirely. White spots left in a tint may be filled up, after it is dry, with the point of the brush, taking care not to apply the color where the paper is already tinted, as that would double its intensity, and make a dark spot. The knife should never be used for erasing on a tinted drawing, as the color *sinks*, and becomes intense, wherever the paper is scratched.

68. After the flat tint it will be necessary to practise the *alternate* or double tint. This consists of two colors, applied alternately, their edges being allowed to melt into each other. For this purpose, two saucers of tint must be prepared, with a brush for each. Begin, as for a flat tint, with one of these colors, at the upper outline of the figure, and having laid on a small portion of that tint, take the brush charged with the other color, and before the first dries, run around its edge with the second, and allow them to blend together, then resume the first tint, blending in the same manner, and so on throughout the space to be filled. It will be observed that the surface of the paper is here treated exactly as if a flat tint were being laid, the color not being allowed to dry anywhere. Each tint is spread upon the white paper, and therefore shows itself pure. The forms of the masses of each color should be varied, and not made in stripes, or spots, but irregularly clouded. The tints in this sign should be light, and equal to each other in strength.

69. For the use of the colors as follows, see *Fig.* 36 :—

Water, a flat tint of pure Indigo, light colored (a).

Sand, a flat tint of Yellow Ochre (b).

Cultivation, a flat tint of Burnt Sienna (c).

Grass Land, or *cleared ground*, a flat tint of green, composed of Indigo and Gamboge, the latter predominating (d).

Uncultivated Land, or *Brushwood*, a double tint of alternate *green*, as for cleared land, and Burnt Sienna, laid on as described in *Par.* 68. This sign may also be made with alternate Green and Crimson Lake, it being the only double tint used (e).

Buildings, and in general all masonry, such as bridges, locks, walls, &c., are tinted with Crimson Lake, and shadowed with a neutral mixture of Indigo, Burnt Sienna, and a little Lake. For this, and all other purposes of light and shade, as in forest, &c., the light is supposed to enter the drawing at the upper left hand corner of the margin, in parallel rays, inclined 45° to the horizon. The shadow of any object will, therefore, surround its lower and right hand outlines. After the shadow has been placed, the outlines must be strengthened, by making it a heavy black line on the sides opposite the light (f). This reinforcing the outline is always the last work on the drawing, and must never be undertaken while any brush-work or tinting remains to be done.

Roads, *streets of towns*, and all portions of the drawing not particularly described, are tinted with Yellow Ochre.

70. In the signs for *Forest and Marsh*, some attempt is made at a resemblance to the things signified, and a more minute description of the method of executing them will be required. For *forest* (*Fig.* 36, g) the ground is first prepared by laying a flat tint of green, exactly as for cleared land. Then with a very hard and sharp lead pencil, or a pen with pale ink, groups and masses of trees are drawn *in outline*, as described in *Par.* 56. Then, with Indigo and Gamboge, mixed in the brush to the same tint as the ground color already laid on, but a little more intense, each tree and mass of foliage is *shaded* on the right hand and lower portion. An orange tint, made of Gamboge and Burnt Sienna, is then touched upon the side *next* to the light. These two tints should just fill up the outline, the green occupying about two thirds of the figure. This finishes the tree. It only remains to add the shadow upon the ground, which is made and laid on as directed in *Par.* 69. For single trees, as in orchards, the shadow is *detached*, but in masses of foliage, it is laid close to the outline (h). The above is merely a formula for this sign, but there is always an opportunity for the taste of the draftsman to produce a good effect, by presenting trees somewhat in the manner of landscape painting.

71. *Marsh*, as in pen-drawing, is composed of a mixture of the signs for water and cleared land, which interlace horizontally, that is, always parallel to the base of the drawing (i). The green tint of marsh is first to be laid in with a brush moderately charged with the color for cleared land. In doing this, attention must be paid to that part left white, which latter must be rather less in quantity than the green, and must resemble it in form, projecting its horizontal points into the green, just as the green projects into the white. The outer limits of a marsh must be made up of an outline of projecting green points. When that is done, a thin shading line must be drawn along the *lower edge* of the green. This is composed of Indigo and Burnt Sienna, and must be confined within the limits of the green tint, and not allowed to touch the white. This finishes the land portion of the marsh. The water is represented with very light indigo, laid on in horizontal streaks of varied width, just filling up the white space without encroaching on the green. The introduction of a single tree here and there assists the good effect of this sign.

72. In painting the trees in forest, and shading the banks of marsh, &c., it is not necessary to mix tints in a saucer. It will be sufficient to rub the three colors, Indigo, Burnt Sienna, and Gamboge, side by side upon the same plate, and with a proper quantity of water in the brush, to mix them to the proper color and intensity, so that they may be applied without lying in drops upon the paper.

73. In regard to the representation of slopes and declivities, the principle heretofore laid down, viz. a steep or a gentle slope being indicated by a darker or lighter color, must be here followed. The tint used to replace that produced by the pen-lines, is composed of Indigo and Burnt Sienna, when the groundwork is green; and when used over sand, or cultivation, a little Lake is added to the mixture in order to neutralize its greenish hue. This shade tint is always laid on after the ground is covered with an appropriate sign, and as no hard edges can be admitted, the following method is used to avoid them (k):—With clean water in the brush, moisten well the paper along the line of the crest of the slope, and before it dries begin in the *lower edge* of the moistened part to lay on the shade of the slope. Proceed down the hill with it, laying it on like a flat

tint, until the lower limit of the slope is reached, when, while the shade tint is still moist, it must be softened off, or blended, with a brush and clean water. If the slope be of great hori zontal extent, its sides may be shaded in successive portions, provided no hard edges are left on the tint, because a slope can never be finished by once tinting, but requires repeated touching, so that the proper depth of shade may be procured, and so that all the detailed variations of declivity may be indicated by corresponding degrees of intensity. Then if the joints of the tint are made in different places each time of going over, they will not show themselves. This method of representing hills is very expeditious, but its effect can be much improved by the addition of light shading lines drawn with the pen, either with pale ink, or a mixture of Burnt Sienna and Indigo.

74. In reference to the general effect of these tinted topographical drawings, two things are to be considered in the colors, viz. the quality of the mixed tints, and the strength or intensity of color. Greens should not be of a *cold* quality, such as is produced by too much blue, but of a lively hue, which is produced by a predominance of Gamboge. As to intensity, everything should be subordinate to the condition of *clearness*; next, the tints must be clean, transparent, and rather light, so as not to mask any of the details of the drawing. They must be of sufficient strength, however, to be readily distinguished from each other at once, and even a very little stronger than necessary so as to allow for fading.

All tints that are much extended ought to be *balanced*, that is, no one ought to obtrude itself upon the eye by its too great intensity. According to the terms of art, everything should be " in keeping," and *spottiness* avoided. Thus forest, brushwood, and cultivation, ought to be kept about equal in strength. Cleared land, marsh, sand, and water, may be made of equal intensity, but all a little lighter than the other signs. Tints that are of small extent, may be a little exaggerated in intensity in order that they may be readily distinguished. Buildings, being objects of small extent, and having a certain importance, require a well marked red tint, shadow, and shaded outline. Villages, with their gardens, orchards, &c., should generally be represented a little stronger in the tints that com-

pose them, than the general tone of the surrounding country (*Fig.* 36, at m).

75. The order in which the tints for the different signs are successively laid on, is a matter for the experience and skill of the draftsman to decide. It is generally thought better, in order to insure a proper balance among them, to begin with the lightest.

76. Cultivated ground is sometimes striped (*Fig.* 36, at c) in the manner of furrows, with lines of red, yellow, blue, or green, lightly laid on, and in various directions.

77. Inclosures are represented as follows. Hedges by green dots, varied in size, for bushes, with the shadows. Masonry or brick walls, by a line ruled with red. Wooden fences, by lines either ruled or hand-drawn with a pen, containing the neutral mixture of Indigo, Burnt Sienna, and Lake (n).

78. After the delineation of all the topographical features heretofore described, the next most important labor of the draftsman is the *lettering* of his map. And in this department his responsibility as to the good appearance of his drawing is very considerable, for nothing so surely detracts from the value of a map, viewed as a work of art, as an awkward and unhandsome style of lettering. To make manuscript letters, imitating type-printing, requires *a great deal* of study and practice, and to proportion the title and other lettering, so as to suit the scale and general effect of a map, is a matter of considerable importance. As far as written directions can be of service in guiding the draftsman in this part of his work, the following rules may assist him: 1st, as to the *time* for lettering. If the map is a *pen drawing*, all letters that fall upon the surfaces of lakes, rivers, &c., or upon the sides of steep hills, or in a forest, should be put upon the drawing before those features are drawn, for it is easier to pass their characteristic lines over the letters, than to draw the letters upon the paper already so occupied. In a *tinted drawing*, the letters are always the last drawn, as a brush cannot be passed over them without blotting. 2d, as to the *size* of the letters. This depends upon two things; the importance of the object described, and the scale of the drawing. The different characters or types of lettering employed, are thus arranged in regard to importance: 1st, *the upright capital;* 2d, *the inclined capital;* 3d, *the upright Ro-*

man, or ordinary small type; and 4th, *the small Italic.* The first of these characters belongs to such objects as large cities, an extensive forest, a bay or gulf, an island, or a considerable mountain or river. These same objects, when they are of less importance, may be described in inclined capitals, as may also a village. A road or canal, in Roman small type, and farms, residences, &c., in Italics. In regard to proportioning the size of the letters to the scale of the drawing, the following table will serve as a general guide, which is all that can be pretended, as an acquaintance with the way in which this proportion is arranged in drawings of merit, will furnish the best rules of practice:—

Scale.		*Height of upright Capitals.*	*Height of small Roman.*
$\frac{1}{600}$	or one in. to fifty ft.	Six-tenths of an inch.	Twelve-hundredths of an in.
$\frac{1}{2640}$	or two ft. to one m.	Four " "	Eight " "
$\frac{1}{5280}$	or one ft. to a m.	Three " "	Six " "
$\frac{1}{10560}$	or four in. to a m.	Two " "	Four " "

The variation in height for each scale, from that of the upright capital to the italic, is gradual, and is regulated as above stated by the importance of the object. The *thickness* of the capitals, in proportion to the size, is *one seventh* of the height of the letter. In lettering a drawing, it is desirable that all the names should be in lines parallel to the base. Rivers, roads, canals, &c., sometimes require the line of the letters to be both oblique and curved; care must be taken, in such instances, to place it so that the words may be easily read without turning the drawing.

79. *The formation* of the letters requires great attention and study. The beginner must copy from good models, all the different kinds of character, and thus acquire a perfect knowledge of the proper proportions and expression of every letter, both large and small. He must then exercise himself by drawing them without a model, in order to acquire manual skill, which he cannot be said to have done until he is able to form all the letters correctly, and give them their proper symmetry

and family resemblance, having for a guide only the two pencil lines which limit their height. In making capitals, each letter must be fairly sketched in pencil, including the thickness of the heavy parts; the outline must then be drawn in ink, with a firm and steady line, and afterwards filled up with the pen. Whether capitals are upright or inclined, it is well to draw a few lines in pencil, parallel to the direction of the letters, which will serve as guides in drawing them. When it is required that one or more words shall occupy a certain place on the paper (as in titles of maps, where the middle points of the lines lie upon a vertical line), find the middle letter or space, and put it in its proper position, then draw the latter half of the word or line. Proceed from the centre to the left, and draw in the first half, by putting in the letters in inverse order. This method often saves repeated trials and erasures. For drawing the small roman and italic letters, the same kind of study with the pencil is at first required; but as the heavy parts of these are made at once, by a bold pressure upon the pen, the operation of making them resembles careful *writing.* As a preparation, three pencil lines are drawn, the lower two to form the upper and lower limits of the ordinary letters, and the upper one to limit the capitals and the tops of the *l*'s, *d*'s, &c. The parts of these letters are of two kinds, viz.: *curved* and *straight*, which should be carefully distinguished from each other. For example, *a*, *c*, *g*, *o*, *s*, &c., are composed entirely of curves. They must be symmetrically drawn, and the width of the ordinary letter must be only a little less than its height. The round part of a *g* does not reach quite to the lower line. The letters *b*, *d*, *f*, *h*, *m*, *n*, *p*, *q*, &c., are composed both of curved and straight parts. The uprights of these letters must be made *perfectly* straight from top to bottom, with a little horizontal return, pointing to the left at the top, and at the bottom to the right. The *m* and *n*, although curved at the top, must be brought down *straight* to the lower line, with a return pointing to the right. These returns must always form a sharp angle with the line of the letter, and not be *rounded.* See *Fig.* 37 for the application of these rules. The letters *i*, *k*, *l*, *v*, *w*, *x*, and *z*, being composed entirely of right lines, care must be taken to keep their elements straight. The beauty of this kind of manuscript depends more upon the

regularity and mutual likeness of the letters, than upon their individual character. In italics, the inclination of the lines must be everywhere the same. As compared with clear roman or italic type, manuscript lettering ought to occupy rather more horizontal space, and always looks better when somewhat extended; crowding it injures its effect very much. The little returns at the top and bottom of the straight parts may also be inclined a little from the horizontal, the left hand extremity of it pointing a little downwards. When executed with freedom and regularity, there is a peculiar beauty in lettering with the pen, which does not depend upon a resemblance to printed work. Finally, the merit of a map as to accuracy even, is not safe from doubt, when, however correctly drawn, its style of lettering marks a want of knowledge or skill in so simple a matter as the formation of the letters of one's language.

80. A map being drawn and lettered, to complete it there are required a border, a descriptive title, and the meridian line, and the scales.

81. First. *The border.* The taste and fancy of the draftsman may sometimes suggest such a composition of lines or figures for this purpose, as will greatly embellish a drawing; but if a plain one is required, the style generally adopted is a double line, one heavy one on the exterior, and one light interior one, the heavy line having the same breadth as that of the blank space between it and the light line. As the map is generally a rectangle, the rule usually followed for proportioning the breadth of the border (including the two lines and the space between them) is, to make it the one hundredth part of the length of the short side of the rectangle. A quill pen, with a very broad point (cut off, not square, but by inclining the cutting instrument towards the body), and without any *split*, is the best instrument for drawing very broad ink lines.

82. Second. *The title.* This may be placed outside the border, if it takes up only one line; but if it requires several, then it must be placed within it. The letters composing the name of the *locality*, which is generally the most important word, should not exceed in height three hundredths of the length of the short side of the border. The letters of other words are varied in size according to the importance of the words they

compose. The execution of the title furnishes another opportunity to enhance by its ornamental character, the beauty of the drawing. It ought to occupy one of the corners of the map, and to have the middle points of all its lines of words or phrases, upon a vertical line. It should state, in small letters, the name of the draftsman, the dates of the surveys, and of the drawing, and under whose direction executed.

83. Third. *The meridian, or north and south line.* This is an indispensable adjunct to every topographical drawing or map. When the extent of country represented is very considerable, it is generally managed so as to make the top of the sheet the north side of the limits. The upright sides of the margin are then north and south lines, and the word "north" may be written outside and above the upper border. But if the shape of the ground included does not conveniently admit of this arrangement, then a meridian line must be determined, and projected upon the map. The importance of this line is evident, as without it no just idea of the situation of the locality with reference to the surrounding country can be obtained; nor could the drawing be compared, or used in connexion with any other map, unless it has this fixed line of direction, which is common to all. The true meridian cannot be determined by the magnetic needle, which does not always point to the geographical pole; but the following is a very easy way of finding it by means of a plummet and a watch. At some point laid down upon the map, suspend a plumb-line over a table, which has been made exactly horizontal. The line should hang from the very extremity of a pointed rod, which should be inclined about 45° to the table, and directed towards the north. The plumb-bob should have a sharp point, which must, as nearly as possible, touch the table. At any two moments equally distant before and after twelve o'clock, say at 9 A.M. and at 3 P.M., mark exactly the extremity of the shadow cast by the rod, and from each of these points draw a line to the point immediately under the plummet. A line bisecting the angle formed by these two lines, will be a true meridian, if the watch indicated the true noon, and has not altered its rate of going between nine and three o'clock. It is easy to prolong the line thus formed, and to project it on the map, either by finding upon its prolongation some other point laid down on

the map; or if there be none, by measuring the angle between the meridian and a line joining the plummet, and some other known point laid down. The meridian may be found without using a watch, by marking the extremities of two shadows of equal length, one cast by the sun before, and the other after noon. It is usual to make the meridian line a conspicuous one, and to ornament its north extremity with some fanciful device, though a simple arrow-head, with the letter N, will answer all the purpose.

84. Fourth. *The scales.* Every drawing should have two scales—one the scale of *spaces*, from which to deduce the degree of declivity, as explained in *Par.* 34, and the other, the scale of *horizontal distances.* They should be carefully measured and drawn upon the map, in any convenient position within the border. For the scale of distances, the length of line measured ought to be between a fourth and a third of the long side of the border.

85. The conventional signs for bridges, roads, &c., and other topographical minutiæ, are exhibited in *Fig.* 34. The nature of each object there represented may, if desirable, be further explained by descriptive lettering; for example, a mill, or factory, being indicated by the proper general signs for such objects, can better be described in letters as a flour-mill, or cloth factory, &c., than by making a different sign for every kind of mill. Numerous signs, differing, as they must, very little from each other, are either a heavy burden to the memory, or are unintelligible without an explanatory table.

OF SCALES.

Before projecting upon a map the data collected by a survey, it is necessary to decide upon some scale of horizontal distances, suited alike to the purposes for which the map is intended, and to the nature and amount of detail that it is required to represent. If the scale is too small, it banishes many details that might be desirable; if too large, it produces an unwieldy drawing. In order that a scale may be a convenient one for use, it is necessary, on the one hand, that the dimensions measured on the ground should be converted, without calculation, or by an easy effort of the mind, into the cor-

responding dimensions on the map; and on the other hand, that the dimensions of the ground should be just as easily inferred from those of the map.

For example, in the scale of one foot to a thousand feet, or $\frac{1}{1000}$, one hundred feet of the ground is represented by one-tenth of a foot on the map, or .1, one hundred and fifty feet by .15, and one hundred and seventy-eight feet by .178, or 178 thousandths of a foot, which bears so close a relationship in its figures, to the distance measured on the ground, that it can be at once taken in the dividers from a scale, which will hereafter be described. On the contrary, a scale is inconvenient for use, when the denominator of its ratio is such a number as $\frac{1}{7920}$, or eight inches to a mile, or $\frac{1}{10560}$, which is six inches to one mile. The long measure used in our country, viz. miles, yards, feet, and inches, has been the means of retaining these arbitrary ratios in use among our draftsmen, but in the service of the United States Coast Survey, the decimal scale is adopted for all maps. The French long measure being expressed entirely in decimals, makes the application of the decimal scale perfectly appropriate; but with us, it is easier to estimate distances by miles, half miles, and yards, than by thousands or hundreds of feet. It is proposed to treat of both these kinds of scales, and to show how arbitrary ratios may be expressed by a diagonal scale of equal parts.

There are two scales to be constructed for a map:—One, the *scale of distances*, which the draftsman puts upon the drawing after it is finished, and which is used only in finding and comparing distances on the paper; the other, the scale of *construction*, intended to furnish the smallest measurements that may be required in projecting dimensions on the drawing.

To construct the scale of distances (*Fig.* 38), draw a right line with the pencil, and supposing, for example, the scale to be $\frac{1}{1000}$, divide it into equal parts, each one tenth of a foot in length. This ought to be done with the graduated edge of a good scale, for though the compasses would give equal divisions, yet we could not be sure that each one was exactly $\frac{1}{10}$ of a foot, or that the sum of ten of these would be exactly equal to one foot. Above these points of division write the numbers 0, 100, 200, 300, &c., to 1000. Then 2000, 3000, &c., at intervals of a foot. Prolong the measured line on the left of the zero,

and lay off the distance of one tenth of a foot, which must be divided into ten equal parts, and numbered in their order from the zero to the left. Put this line in ink, and draw beneath it a heavy line, say two-hundredths of an inch wide, and at a distance below the first line equal to its width. This heavy line should not pass to the left of the zero of the scale. Then with a right-line pen, rule the divisions before measured off, drawing them perpendicular to, and across both lines of the scale. If it should be required, for example, to take from this scale a distance of 180 feet, place one foot of the dividers upon the point marked 100, and carry the other beyond the zero to the left, until it comes to the division marked 80. The compasses will then include the required distance. Any odd number of feet will be found by supposing each of the small spaces on the left of the zero to be divided into ten equal parts, and placing the foot of the compasses accordingly.

The scale of construction (*Fig.* 39), being intended to express smaller dimensions than the scale of distances, which, it will be observed, shows nothing smaller than hundredths of a foot, is constructed as follows. After having subdivided and numbered a right line as before, let fall perpendiculars from every point of division, then draw ten other lines below, and parallel to the first, equidistant from each other. This equal spacing may be two tenths of an inch, or less, or more, provided it be constant for each space. Now in the space of the $\frac{1}{10}$ of a foot which lies on the left of the zero, draw the diagonal lines, as in the fig., by joining the first division on the left of zero (the point c) with the point b, and drawing through the points 20, 30, 40, &c., lines parallel to $b\ c$. From this construction, and the properties of similar triangles, it is evident that that part of the line AB (the second line from the bottom), which is included between the sides of the triangle $o\ b\ c$, is equal to one-tenth of $o\ c$, the base of the triangle; that the corresponding part of the third line, CD, is equal to two-tenths of the base, $o\ c$, of the fourth line, it is equal to three-tenths, and so on. But the base, $o\ c$, of the triangle is the $\frac{1}{100}$ part of a foot, therefore the parts of the horizontal lines intercepted by its sides are, respectively, $\frac{1}{1000}$, $\frac{2}{1000}$, $\frac{3}{1000}$, &c., of a foot, which fact is expressed by the figures 1, 2, 3, 4, &c., upon those lines. If it be required to find from this scale, a distance of 277 feet,

place one foot of the dividers at d, on the vertical line numbered 200, and upon the horizontal line numbered 7, then place the other foot at e at the intersection of the same horizontal line with the *diagonal* numbered 70, and the distance will correspond to 277 feet This is called the *diagonal scale of equal parts*, and a scale thus constructed is applicable to all decimal ratios, the numeration only changing with the ratios. If the distances are expressed in other terms than in feet, the top line of the scale must be divided according to those terms. For example, if the scale were one inch to a hundred feet, the upper line of the scale must be divided into inches, and the inch on the left of the zero into tenths. Then the ten horizontal lines, and the diagonals, will express hundredths of an inch, or one foot on the ground. If, instead of using brass or ivory scales, the draftsman makes the scale himself, which is to be preferred, it must be made upon a piece of the same paper as the drawing, so that there will be no unequal variations in the scale and drawing, caused by heat or moisture, which affect different specimens of paper very differently.

Other ratios may be expressed on the diagonal scale by the following method: Suppose it is required to construct a scale of 24 inches to 1 mile, which shall be capable of measuring any odd number of feet. The ratio of this scale is $\frac{1}{2640}$, that is, one foot of the drawing corresponds to 2640 feet on the ground. Then one inch of the drawing (and of the scale to be constructed) corresponds to $\frac{2640}{12}$, or 220 feet, and one-tenth of an inch to 22 feet. Draw the line of the scale as before (*Fig.* 40), divide it into inches, and number the points of division from the zero to the right, 0, 220, 440, 660, 880, &c. Divide the inch on the left of the zero into ten equal parts, and number them 0, 22, 44, 66, 88, 110, &c., from the zero towards the left. Now draw *eleven* parallel and equal-spaced lines below the first one, and draw the diagonals in the space on the left of the zero. It is evident, from an inspection of this scale, that the parts successively cut off upon the horizontal lines by the sides of the triangle *obc* are, commencing from the line next to the lowest, respectively equal to $\frac{1}{11}$, $\frac{2}{11}$, $\frac{3}{11}$, &c., of 22 feet, or the base of the triangle, which is one-tenth of an inch, or in other words, they are respectively equal to 2, 4, 6, 8, 10, &c., feet. Single feet are found by placing the compasses midway between the

horizontal lines. This scale is very much inferior, in point of convenience, to the decimal scale, on account of the complicated numbers which express its divisions, the addition and subtraction of which cannot be readily effected by a mere mental operation. Other scales may be constructed as follows:—12 *inches to a mile, or* $\frac{1}{5280}$: Divide the line into inches, and number the divisions 0, 440, 880, &c. Divide the inch on the left of the zero into *eight* equal parts, each of which will be 55 feet; draw *eleven* lines below and parallel to the first line, and draw the diagonals. The parts cut off on the horizontals by the sides of the triangle, will be successively 5, 10, 15, 20, &c., feet. 8 *inches to* 1 *mile, or* $\frac{1}{7920}$: Divide the scale into inches, and the left hand inch into *ten* parts. Draw *eleven* horizontal lines below, and the diagonals; the parts then cut off will be 6, 12, 18, 24, &c., feet successively. 6 *inches to* 1 *mile, or* $\frac{1}{10560}$: Divide the line into inches, and the left hand inch into *eight* equal parts. Draw *eleven* horizontals. The distances cut off will be 10, 20, 30, 40, &c., feet. 4 *inches to* 1 *mile, or* $\frac{1}{15840}$: Divide the line into inches, and the left hand inch into eight parts. Draw *eleven* horizontals, and the distances will be successively 15, 30, 45, &c., feet. The subdividing qualities of all these scales may be increased by augmenting the number of the horizontal lines. For example, in the scale of 6 inches to 1 mile, if there were 22 horizontals instead of eleven, the successive distances would be 5, 10, 15, 20, &c. If the horizontals were doubled in number in that of 8 inches to a mile, the distances would be 3, 6, 9, 12, &c., and if they were trebled in the scale of 4 inches to a mile, or made 33 in number, we should have distances of 5, 10, 15, 20, &c., feet.

The reason for drawing eleven horizontals in some of the above described scales is, that the number 11 is an exact multiple of the number of feet represented by the divisions of that part of the line on the left of zero. A great multiple ought to be selected, so as to have a convenient number of horizontals. To take one more example. 5 *inches to* 1 *mile, or* $\frac{1}{12672}$: Here one inch of the scale corresponds to 1056 feet, or $\frac{1}{8}$ of an inch to $\frac{1056}{8} = 132$ feet. We divide the inch on the left of the zero into eight parts, and for the number of horizontals we seek the largest desirable factor of 132, which may be either 22 or 12. If we draw 22 horizontals, the distances given by

the diagonals will be 6, 12, 18, 24, &c., but if 12 horizontals are used, they will give distances of 11, 22, 33, 44, &c., feet. It is evident, also, that the inch on the left of zero must be so divided, as that the number of its equal parts must be a multiple of the number of feet corresponding to the inch, as above. 1056 is a multiple of 8, and not of 10, hence we divide the inch into eighths, each of which is 132 feet.

Unless these multiples can be found, the scale cannot be made to express whole numbers by its smaller divisions, which is a great defect. It is to be hoped that the decimal scale and ratio will eventually be adopted.

In adopting the scale to the uses of a map the following general proportions may be observed: A map constructed on a scale of half an inch to a mile, or $\frac{1}{126720}$, will admit the representation of all towns, villages, main roads, the principal cross-roads, and every considerable mountain and stream. On a scale of one inch to a mile, or $\frac{1}{63360}$, besides these, farms, woods, isolated buildings, every stream of 600 feet in length, and every hill of a hundred feet in height, can be represented. On a scale of two inches to a mile, or $\frac{1}{31680}$, the various features of the ground can be clearly and accurately presented, also every stream of not less than 300 feet in length, every pond of more than 50 feet broad, besides all roads, isolated buildings, &c. The scale of six inches to a mile, or $\frac{1}{10560}$, is well suited for the complete delineation of a country. Scales for projecting experimental surveys for civil purposes very seldom exceed twelve inches to a mile, or $\frac{1}{5280}$. Larger scales than these are only used in proportion to the amount of detail required. The decimal scales corresponding nearly to these are $\frac{1}{120000}$, $\frac{1}{60000}$, $\frac{1}{30000}$, $\frac{1}{10000}$, and $\frac{1}{5000}$. The smallest publication scale of the U. S. Coast Survey is $\frac{1}{80000}$, which is also the scale of the new map of France.*

OF MERIDIANS AND PARALLELS OF LATITUDE.

In a very extended survey, where latitude and longitude are

* The reader is referred to the beautiful maps of the United States Coast Survey, and to the very elegant detailed map of France, both in course of publication, as admirable illustrations of topography.

considered, it is necessary to project the meridians and parallels upon the map. If the portion of the country included in the survey does not exceed one hundred miles in length and breadth, no appreciable distortion of outline will be occasioned by drawing straight lines at right angles with each other in order to represent them. But if a greater extent of country is comprehended, it will be necessary to show the convergence of the meridians towards the poles, and the consequent diminution in the length of a degree on the higher and smaller circles of latitude. The following are two practical methods for determining these lines in both cases:—

Suppose, as in the first of these cases, that A B C D (*Fig.* 41) is the boundary of a topographical map, upon which the meridians and parallels of latitude for every tenth minute are to be drawn. It is necessary, for this purpose, that the latitude and longitude of one point in the map should be known, and the direction of the true meridian passing through it determined. Suppose the longitude of the point *a* to be 4° 23′ West, and its latitude 42° 18′ North, its meridian being in the direction SN. Draw through *a* the line WE at right angles with the line SN. This will be the parallel of latitude passing through the point *a*. Since the meridian for every tenth minute is required, the first one West of *a* to be determined is that of 4° 30′, which is seven minutes West of *a*. The first one to be determined on the right is that of 4° 20′, which is three degrees East of *a*. The degrees of longitude expressing these distances must be converted into miles or yards, and then laid down upon the map according to the scale of distances. For this purpose the following tables are used. Table 1 gives the different lengths of a degree of longitude, in terms of geographical miles, and table 2 the same, in terms of statute miles:—

TABLE I.

Showing the Length of a Degree of Longitude for every Degree of Latitude in Geographical Miles.

Lat.	*Geographical Miles.*	*Lat.*	*Geographical Miles.*	*Lat.*	*Geographical Miles.*	*Lat.*	*Geographical Miles.*
0	60.00	23	55.23	46	41.68	69	21.51
1	59.96	24	54.81	47	41.00	70	20.52
2	59.94	25	54.38	48	40.15	71	19.54
3	59.92	26	54.00	49	39.36	72	18.55
4	59.86	27	53.44	50	38.57	73	17.54
5	59.77	28	53.00	51	37.73	74	16.53
6	59.67	29	52.48	52	37.00	75	15.52
7	59.56	30	51.96	53	36.18	76	14.51
8	59.40	31	51.43	54	35.26	77	13.50
9	59.20	32	50.88	55	34.41	78	12.48
10	59.08	33	50.32	56	33.55	79	11.45
11	58.89	34	49.74	57	32.67	80	10.42
12	58.68	35	49.15	58	31.79	81	9.38
13	58.46	36	48.54	59	30.90	82	8.35
14	58.22	37	47.92	60	30.00	83	7.32
15	58.00	38	47.28	61	29.04	84	6.28
16	57.60	39	46.62	62	28.17	85	5.23
17	57.30	40	46.00	63	27.24	86	4.18
18	57.04	41	45.28	64	26.30	87	3.14
19	56.73	42	44.95	65	25.36	88	2.09
20	56.38	43	43.88	66	24.41	89	1.05
21	56.00	44	43.16	67	23.45	90	0.00
22	55.63	45	42.43	68	22.48		

TABLE II.

Showing the Length of a Degree of Longitude for every Degree of Latitude in English Statute Miles.

Lat.	*Eng. Miles.*	*Lat.*	*Eng. Miles.*	*Lat.*	*Eng. Miles.*	*Lat.*	*Eng. Miles.*
0	69.2000	23	63.6986	46	48.0705	69	24.7992
1	69.1896	24	63.2177	47	47.1944	70	23.6678
2	69.1578	25	62.7167	48	46.3038	71	22.5294
3	69.1052	26	62.1963	49	45.3994	72	21.3842
4	69.0312	27	61.6579	50	44.4811	73	20.2320
5	68.9363	28	61.1001	51	43.5489	74	19.0743
6	68.8208	29	60.5237	52	42.6037	75	17.9103
7	68.6845	30	59.9293	53	41.6453	76	16.7409
8	68.5267	31	59.3162	54	40.6751	77	15.5665
9	68.3481	32	58.6851	55	39.6917	78	14.3874
10	68.1489	33	58.0360	56	38.6959	79	13.2041
11	67.9288	34	57.3696	57	37.6891	80	12.0166
12	67.6880	35	56.6852	58	36.6705	81	10.8250
13	67.4264	36	55.9842	59	35.6408	82	9.6306
14	67.1448	37	55.2659	60	34.6000	83	8.4334
15	66.8424	38	54.5303	61	33.5489	84	7.2335
16	66.5192	39	53.7788	62	32.4873	85	6.0315
17	66.1760	40	53.0100	63	31.4161	86	4.8274
18	65.8134	41	52.2259	64	30.3352	87	3.6219
19	65.4300	42	51.4253	65	29.2453	88	2.4151
20	65.0265	43	50.6094	66	28.4464	89	1.2075
21	64.6037	44	49.7783	67	27.0385	90	0.0000
22	64.1609	45	48.9313	68	25.9230		

If the scale of the survey is in statute miles, the length of a degree of longitude, in latitude 42°, will be found in Table II. This length is 51.4253. To find the distance from *a* to the first meridian on the left, which is seven minutes west of *a*, the following proportion is stated:—

60′ : 51.4253 : : 7′ : the required distance, or 5.9996 miles which must be laid off from *a* to the *left*, and the meridian drawn through the point *b*, so found, parallel to NS. This will be longitude 4° 30′. To find the distance from *a* to the first meridian on the right, which is three degrees East of *a*, the following is the proportion to be stated :—

60′ : 51.4253 : : 3′ : the required distance, or 2.5713 miles, which must be laid off from *a* to the *right*, and the meridian drawn as before through the point *c* so found. Other meridians are drawn parallel to NS, and at a distance from each other equal to *bc*. The parallels of latitude for every tenth minute are determined in the same way, for the parallel WE, being 42° 18′, the first one North, or 42° 20′, will be two degrees North of *a*, and the first one South, or 42° 10′, will be eight degrees South of *a*. The following proportions may then be stated, noting that the length of *every* degree of latitude is sixty geographical miles, or 69.2 statute miles :—

60′ : 69.2 : : 2′ : the distance North of *a*, or 2.3 miles.

60′ : 69.2 : : 8′ : the distance South of *a*, or 9.2 miles.

Parallels of latitude are drawn through the points *d* and *e*, thus determined, and others parallel to them, and to WE, at a distance apart, equal to *de*. The above fractional parts of miles may be reduced to feet or yards by multiplying the decimal fraction by 5280 feet, or 1760 yards.

In the second case, or where the survey covers a greater extent of surface than a hundred miles in length and breadth, the meridians must converge, and the parallels of latitude must conform to them. The following is a simple construction for this purpose, by which any portion of the earth's surface, measured by degrees, is represented by a similar portion of the map. Suppose the survey to lie between longitudes 1° East and 7° West, and 36° and 45° of North latitude, comprising eight degrees of longitude, and nine degrees of latitude. Let *a* (*Fig.* 42) be a point near the middle of the map, whose latitude is 40° 30′ North, and whose longitude is 3° 20′ West. It is required to draw meridians and parallels for every degree. Draw through *a* the vertical line N S for the meridian passing through that point. Find in miles (as before) the distances from *a* to latitude 41° North, and to latitude 40° North. These distances—taking 69.2 miles for a degree of latitude—are 34.6

miles above and below a. The points b and c are thus determined, and are points on the parallels of 40° and 41° North latitude, and the distance $b\,c$ is one degree of latitude, which must be laid off on the line N S, above and below b and c, until the whole extent of survey included by N S is marked off in degrees. Determine, as in the first case, the meridians 3° and 4° west, by laying off in miles the lengths of 20′ to the right, and 40′ to the left of N and S, which are respectively situated in 45° and 36° of North latitude. These distances are, at N, by Table II, N d=16.3437 miles, and N e=32.6874 miles, which are laid off on $e\,d$ perpendicular to N S. At S, draw $h\,g$ perpendicular to N S, and lay off from the table the distances 18.6614 miles to the right, and 37.3228 miles to the left. Then $e\,d$ and $h\,g$ will be each one degree of longitude, corresponding to the latitudes of 45° and 36° north. Draw the lines $e\,h$ and $d\,g$, and through the points of division on N S draw lines perpendicular to N S, and they will divide $e\,h$ and $d\,g$ into degrees of latitude. The figure $e\,h\,g\,d$ will then be a projection of nine degrees of latitude, and one degree of longitude. Other meridians may be thus determined:—With the diagonal distance $e\,g$ or $d\,h$, as a radius, and from g and h as centres, describe the arcs $i\,k$ and $l\,m$, and from d and e as centres, with the same radius, describe the arcs $n\,o$ and $p\,q$. Now, from the points d and e as centres, and with $e\,d$ as a radius, draw two arcs intersecting $i\,k$ at r, and $l\,m$ at s, and from h and g as centres with $h\,g$ as a radius, draw arcs intersecting $n\,o$ at t, and $p\,q$ at u. Join r with t, and s with u, and divide right lines $s\,u$ and $r\,t$ into degrees of latitude, each equal to $b\,c$, and we shall have the meridians and parallels for 2° and 5° west longitude. Determine the other meridians of the survey in the same manner. The parallels of latitude may be composed of straight lines from one meridian to another, or, their points being determined on the meridians, a *curve* may be drawn through all those points having the same latitude.

OF THE METHOD OF PROJECTING HORIZONTAL CURVES, FROM THE LEVELS OF CERTAIN POINTS DETERMINED BY SURVEY.

In surveying the ground, for the purpose of tracing upon its plan the horizontal curves, the points whose levels are determined must be sufficiently numerous and close together, to admit, without sensible error, of the supposition that the slope of the ground between them is uniform. The following method proceeds upon this supposition. Let A and C (*Fig.* 43), be two points on the profile of the ground, and let the horizontal distance (AB) between A and C be fifty feet.

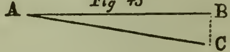

Fig 43

Let the difference of level between A and C, as determined by survey, be ten feet, C being the lowest point. It is required to find, upon A C, the points in which horizontal planes, drawn one foot apart, and commencing at A, will intersect A C. The following proportion will discover this :—

As the total fall from A to C is
To the horizontal distance A B from A to C,
So is any partial fall from A towards C
To its corresponding horizontal distance from A.

Now A B is 50 feet, and the total fall from A to C is 10 feet, then for a partial fall of one foot we shall have 10 ft. : 50 ft. : 1 ft. : 5 ft., or the horizontal distance from A to that point of A C, which is one foot below A : and by laying off 5 feet from A towards C, we shall have the intersection of the one-foot plane with a line of the ground. Again, 10 : 50 :: 2 : 10, which gives ten feet from A, for the point of intersection of the two-foot plane with A C : and so on for the other planes. By marking out upon the ground squares, or triangles, whose sides are of equal and convenient length, determining the levels at all the intersections, and reducing all the levels so that they may be referred to one point (a horizontal plane drawn through such a point is called the *plane of reference*, and the levels so reduced are called *references*), we can, by the above method, find, upon every line, the intersections of any horizontal planes.

But the references, as obtained by the instrument, are

scarcely ever expressed in whole numbers; and whereas it is desirable that the planes should be passed at whole numbers of feet apart, the labor of stating a proportion to calculate every point becomes considerable. This is obviated by the following convenient mechanical method,* by which the proportions, instead of being stated in figures, are presented in lines, by means of the properties of similar triangles.

Let 1, 3, 9, 7 (*Fig.* 44), be a portion of ground, projected on a scale of 50 feet to an inch. It is 100 feet square, and is subdivided into four squares, of 50 feet sides. Let the references of the points 1, 2, 3, &c., be respectively 8.10, 6.30, 7.25, &c., as indicated in the figure. These levels are expressed in feet, and are *referred* to a horizontal plane 2.5 feet above the point 5; which is the highest point of the ground. It is required now to trace the intersections of horizontal planes, which shall be 3, 4, 5, 6, &c., feet below the *plane of reference.* Let us begin with the line 5, 6, Draw the line A B (*Fig.* 45), equal to the line 5, 6, or, according to the scale, fifty feet in length. Then let fall from A & B, two perpendiculars, A D & B C. Divide these perpendiculars into equal parts, say, each one tenth of A B, and join the opposite points of division, forming the ladder-like figure A B C D. Number the horizontal lines from B downwards, in quarters of unity, viz.: .25, .50, .75, 1, 1.25, 1.50, &c., &c., so as to include the greatest number of feet the ground will probably descend, from station to station. In the present case 7.50 will suffice. Cut from a piece of stiff paper a narrow strip like E F, making the edge E F accurately straight. Fasten the line E F to the point B, by means of a fine needle, so as to conceal as little as possible of the corner at B, and the instrument is ready for use. Beginning at the central point, 5 (*Fig.* 44), it will be observed that the three-foot curve is .5, or half a foot below it; the four-foot curve is 1.5 feet below it; the five-foot 2.5, and the six-foot 3.5 below it. The total fall from the point 5 to the point 6, is 6.50–2.50, or 4 feet. Then the edge E F (*Fig.* 45), of the strip must be placed so that the line E F will be drawn from B to G, on the horizontal line marked 4, corresponding with the difference of level

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* **Industrial Drawing. D. H. Mahan.**

between stations 5 & 6. The strip must be secured in this position by a pin near E. Now from station 5, the first partial fall we wish to find, is from *reference* 2.50, to the three-foot curve, or .5 of a foot.

The proportion is,—Total fall from sta. 5 to sta. 6, is to
Distance from sta. 5 to sta. 6, as
Partial fall, is to the distance required.

or, by the instrument, A G, (or B H) : H G :: B i : i k. Hence, to find the horizontal distance corresponding to the partial fall of .5, we have only to measure on the horizontal marked .50, its length included between B H and E F (*Fig.* 45), and lay it off on the line 5, 6, (*Fig.* 44), from 5, towards 6. This will be a point of the three-foot curve. The next point, that of the four-foot curve, is 1.5 feet below station 5. Take the length of the line marked 1.50, included between B H & E F, and lay it off from 5, towards 6. The five-foot curve lies 2.5 feet below station 5 ; then we take the part of the line 2.5 included between B H & E F, and lay it off as before. The six-foot curve being 3.5 feet below, we measure and lay off a similar part of the line marked 3.5. This finishes the division of the line between sta. 5 to sta. 6, and gives points of the three, four, five, and six-foot curves; which must be marked (3) (4) (5) (6). Points of the curves on other lines are determined in the same manner. For example, from sta. 5 to sta. 4, the total fall is 6.50 feet. Set the edge E F, from B to L, on the line marked 6.50, and measure and lay off successively from 5 towards 4, the parts included between E F & B M, of the lines marked .50, 1.50, 2.50, 3.50, 4.50, 5.50. The station 4, having a *reference* of 9 feet, is itself a point of the nine-foot curve. Find the points thus, upon every line of the figure, and draw the curves through the points so determined, taking care to give them their proper curvature from point to point. If the total fall from station to station is expressed in smaller fractions than .25, as for exam ple from sta. 5 to sta. 2, where it is 3.80; then the line E F must be placed at a point between 3.75 and 4, but nearer to 3.75 : or else the line B C may be divided and numbered, so as to show smaller fractions than $\frac{1}{4}$.

In case the great irregularity of the ground should require intermediate levels and references, a distance must be laid off from B towards A, making N B equal to the horizontal distance

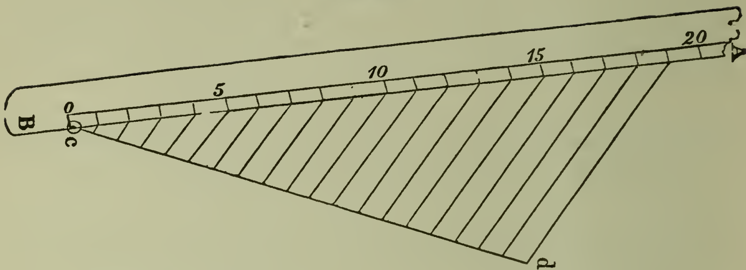
between these secondary points and the primary ones, and the line N O drawn, and used for these, instead of the line A D, which latter is used for all the regular distances of the survey

OF THE METHOD OF PROJECTING HORIZONTAL CURVES OF THE GROUND UNDER WATER.

In surveying a harbor, or any extensive body of water, flag-buoys are stationed at convenient points, and their positions, their distances from each other, and from some points on the shore at the water line, accurately surveyed and projected on the map. Soundings are taken along these connecting lines keeping the intervals of the soundings exactly equal between any two of the stations, though they may vary for different lines. In order to determine the curves of the bottom, it is necessary to distribute the soundings of each line, equally throughout its length. Suppose (*Fig.* 34, "soundings") the line between the two buoys to be one of the projected lines, and that its length is 630 feet. The number of recorded soundings corresponding to that line is 22, including the soundings at the buoys. This will give 21 intervals between the soundings. Then the line must be divided into 21 equal parts, of 30 feet each. Mark the points of division on the line, and write opposite to each point its corresponding sounding. The points of any desired curve may now be found :—for example, the six-foot and nine-foot curves will pass through the points 6 and 9, the twelve-foot and fifteen-foot curves will pass midway between the points 11½ and 12½, and 14½ and 15½, respectively. In the same manner other lines of soundings may be divided, and points of the curves determined. Through all the points so found, the curves are drawn, after which they are numbered (as in the figure) at a sufficient number of places.

The following easy method of dividing a line into any number of equal parts, will save the labor of measuring, or dividing by trial. Cut a strip of drawing paper, the edge A B of which (*Fig.* 45½) is graduated in equal divisions. If it be required to divide the line *c d* into nineteen equal parts, place the strip so that its edge A B shall make a convenient

Fig. 45½.



angle with $c\ d$, and so that the zero of its graduation shall coincide with the point c. Secure the strip in this position. Now join the point d with the point 19 of the scale A B, and draw, *parallel* to 19 d, lines through all the inferior points of the graduation, and these lines will cut $c\ d$ into nineteen equal parts.

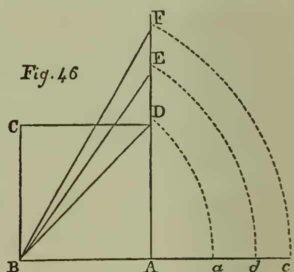
PROBLEMS CONNECTED WITH THE REDUCTION, ENLARGING, AND COPYING OF MAPS OR PLANS.

Problem I.

To Construct a Square that shall be a Multiple of any given Square.

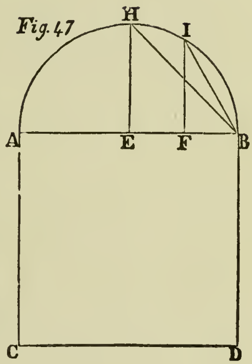
Let A B C D (*Fig.* 46) be the given square, and let it be required to construct a square that shall contain 2, 3, 4 &c., times its surface. Draw the diagonal B D, and make B a equal to B D—then the square described upon B a, will be double the square A B C D. Lay off A E, equal to B a, and draw B E, then the square described upon B E, or B b, will be three times the square A B C D. In the same manner, lay off A F equal to B b, and the square described upon B F, or B c, will be four times the square A B C D, and so for any multiple of the square A B C D.

Fig. 46



Problem II

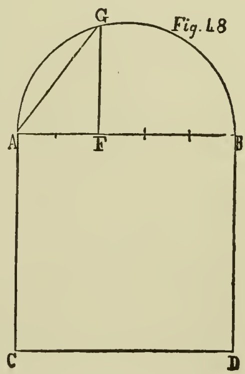
To Construct a Square that shall be equal to ½, ¼, &c., of any given Square.



Let A B C D (*Fig.* 47) be the given square. On A B, as a diameter, describe the semi-circle A H B, and erect, at the centre E, the perpendicular E H. Draw B H, and it will be the side of a square equal to one-half of A B C D. Lay off F B, equal to one-fourth of A B, and erect the perpendicular F I, then the square described upon I B will be equal to one-fourth of A B C D. In the same manner, a square may be constructed, equal to any part of A B C D.

Problem III.

To Construct a Square that shall be in any Proportion to a given Square.

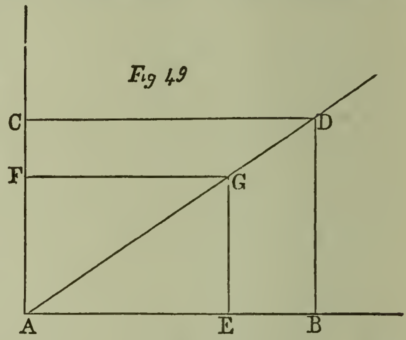


Let A B C D (*Fig.* 48) be the given square. It is required to construct a square which shall be to A B C D as 2 is to 5. Upon the side A B as a diameter, describe the semi-circle A G B, and divide the line A B into five equal parts. At the second point of division, erect the perpendicular F G, and join A G—the square described upon A G will be to the given square A B C D as 2 is to 5.

Problem IV.

To Construct upon a given Base, a Rectangle, which shall be similar to a given Rectangle.

Let A E F G be the given rectangle. It is required to construct upon the base A B, one that shall be similar to A E F B. Prolong A E, and lay off the given base from A to B. Draw the diagonal A G, and prolong it indefinitely. Erect a perpendicular to A B, at B, and at the point D, where it intersects the prolonged diagonal, let fall D C, perpendicular to A F produced. Then A B C D will be similar to A E F G. All rectangles having their diagonals in the same line are similar.



ON THE METHOD OF DRAWING BY THE USE OF REFERENCES, AND PROJECTIONS ON A SINGLE PLANE OF PROJECTION.

Definitions.

Projections. The projection of *a point* upon a horizontal plane is that point of the plane directly beneath the point. Hence to project a point upon a plane, is to let fall from the point, a perpendicular to the plane, and the foot of that perpendicular is the projection of the point.

The projection of *a line* is made up of the projections of all its points. In the case of a straight line, the lines which project its points pass through the given line, and are parallel to each other. They constitute a plane. This plane is called *the projecting plane* of the line. Its intersection with the plane on which the projection is made is a straight line, and is the projection of the line. Hence if the projections of two points of a straight line are known, the projection of the line can be drawn.

The plane on which the projection is made, is called the *Plane of Projection.*

The lines which project points are called *Projecting lines*, and the planes which project lines are called *Projecting planes.*

The projection of a *curved line* will in general be a *curve;* for the projecting lines of its points will not constitute a plane, but a cylindrical surface, which will intersect the plane of projection in a curve.

In what follows, the plane of projection will be taken *horizontal*, and all objects represented are supposed to lie *above* that plane.

Plane of reference. The horizontal plane of projection, above described, is the Plane of reference.

Reference. The numbers expressing the heights of points above the plane of reference are called the *references* of the points, and are written on the drawing, near to the projections of the points.

Line of slope. The line of greatest descent of any plane or surface, is called *the line of slope* of that surface. Also the

line joining any two points in space which are unequally distant from the plane of reference, is a line of slope.

Scale of slope. Any equal divisions, marked upon a line of slope, indicating the *amount of descent* corresponding to any horizontal measurements, constitute *a scale of slope.*

Horizontal scale. A scale of equal parts, indicating the relative distances and positions of points situated in the plane of reference, is called *the horizontal scale.* For each drawing there is but *one* horizontal scale; but there may be as many different scales of slope as there are various inclinations of lines or surfaces represented, and there must evidently be an established relation between every scale of slope and the horizontal scale.

From the above definitions it will be seen that *a point* is determined in space when we know *its projection* on the plane of reference, and its height above that plane:—that is, *its reference.* Also, that *a line* is determined when we know 1st, *its projection*, *its scale of slope*, and the *reference* of *any point* of the line, or 2d, its *projection* and the *references* of any *two of its points.* *A plane* is determined when we know the references of at least three of its points; or when its scale of slope is known.

A few Problems, illustrating the application of the simple principles above defined, will explain the use of this method of representing points, lines, and surfaces, by means of one projection only. It is chiefly useful where (as in Topographical drawing) the heights to be represented are quite small in comparison with the lateral or horizontal extent of the surface.

Any convenient scale of equal parts may be assumed as the Horizontal scale of the drawing. For the following Problems a scale of $\frac{1}{300}$ will be used. Let the unit of length be *a yard;* then the unit of the scale being *one foot*, the scale may be called a hundred yards to the foot, and one-tenth of a foot will represent ten yards.

Problem I.

To represent the positions of points in space.

It is evident from the Definitions, that for this purpose it is necessary only to establish the positions of points relatively to

each other in the horizontal plane, and to write near to each point so established, its *reference*, when its position in space will be fully determined.

Problem II.

To project a right line.

Since the line is determined when any two of its points are known, and since the projection of the line must pass through the projections of its points, it is necessary only to draw the line through two of its established points, and to write their references near those points.

Problem III.

To determine the scale of slope of a given line.

The projection of the line, and the references of two of its points being given, as in Prob. II., let the line in space be divided into any number of equal parts: the projections of those equal parts will evidently be equal to each other. The differences of reference of successive points of division will also be constant. Now, if the line in space be so divided that this constant difference of references shall be a certain unit, say one yard; the divisions on the projection of the line will form a scale of slope, showing the proportion between the slope and the length of the projection. Each point of division on the projection of the line is marked with its reference, and the divisions may be subdivided. Let AE (Fig. 50) be the given line; AP and ER the projecting lines, upon the plane of reference PR, of the given points A and E. Let the reference AP be 9.4 yards, and ER be 5.8 yards. Let the length PR, between the projections of A and E, be 16.2 yards. From the scale of the drawing construct the trapezoid APRE, and through E draw DE, parallel to PR. Lay off AB = 1 yard, and draw BC parallel to PR. Then we have the point C, the reference of which differs from that of the point A by one yard. To determine the value of BC (or PQ), we have

$$AD : DE :: AB : BC.$$

whence

$$BC = \frac{DE \times AB}{AD};$$

or, substituting the numerical values as above,

$$BC = PQ = \frac{16.2 \times 1}{9.4 - 5.8} = \frac{16.2}{3.6} = 4.5 \text{ yards.}$$

The references of the points A and C are respectively 9.4 and 8.4; and to find the projection of a point M, whose reference shall be 9, we divide P Q into ten equal parts, and lay off from P towards Q four of those parts, and we have the point S, whose reference is 9. Taking 4.5 yards from the horizontal scale, and laying it off successively on the right and left of S, calling S 9, the scale of slope is completed, and can be transferred to its line on the drawing.

The length in space of any portion of a line may be found from its references by means of the formula

$$AE = \sqrt{AD^2 + DE^2}$$

by substituting the numerical values. For example (Fig. 50),

$$AE = \sqrt{(16.2)^2 + (3.6)^2} = \sqrt{275.4} = 16.6;$$

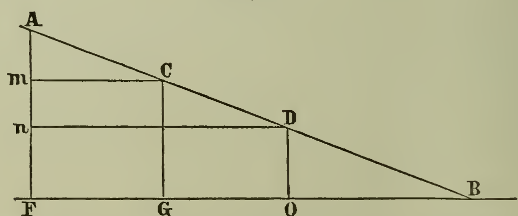
and so for the length between any other two points.

Problem IV.

To determine the slope of a line, having given its projection and the references of two of its points.

The slope of a line is the angle which it makes with a horizontal line. The true slope is the angle that a line makes with its projection on the plane of reference: for every other angle made by a horizontal line with the given line is greater than that made by a line with its projection.

Fig. 51.

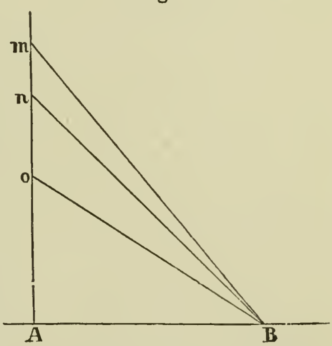


To find an expression for this angle, let AB (Fig. 51) be the

given line, and FB its projection. The angle ABF is the slope. If any number of perpendiculars are let fall from any points, as A, C, D, &c., it is evident, from the similarity of the triangles formed, that the ratios $\frac{AF}{FB}$, $\frac{CG}{GB}$, $\frac{DO}{OB}$, &c. are all equal. The same may be said of the ratios $\frac{Am}{mC}$, $\frac{An}{nD}$, &c.: so that this constant ratio may be taken to represent the angle, or slope. Am is the difference of the references of A and C, and mC is the length of projection between A and C: and if Am = 3, and mC = 7, the slope of the line AB may be represented by the ratio $\frac{3}{7}$. This is called "*a slope of* 3 *upon* 7," that is, 3 perpendicular, to 7 horizontal. If now the *horizontal measurement*, or the base, be *unity*, the perpendicular itself will represent the slope.

Thus in Fig. 52, AB = 1. Let Ao = $\frac{5}{8}$, An = 1, and Am = $1\frac{1}{5}$, or $\frac{6}{5}$: then oBA will be $\frac{\frac{5}{8}}{1}$, or "5 upon 8," nBA will be $\frac{1}{1}$, or 45°, and mBA will be $\frac{\frac{6}{5}}{1}$, or 6 upon 5: proper fractions indicating a slope less than 45°, and improper fractions a greater.

Fig. 52.



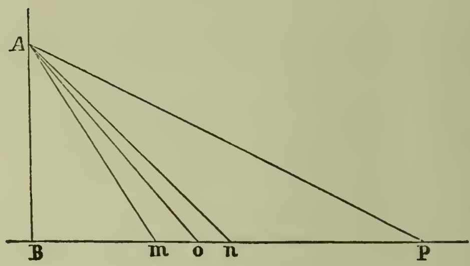
In general, the slope of a line is expressed by dividing the difference of reference of any two of its points by the number expressing the length of the projection between those points.

This ratio is called the *tangent of the angle;* and if it should be desirable to know the number of degrees, minutes, and seconds, in the angle, the fraction must be changed to a decimal of five or six places; and in a Table of Natural tangents the degrees, &c. will be found opposite to that decimal (or mixed number, as the case may be).

If the *difference of the references* of two points be *unity*, the

expression for the slope is derived in the same way, but will be the reciprocal of the former expression, and will depend on

Fig. 53.



the length of projection. In Fig. 53, AB = 1; Bo = .833 or $\frac{5}{6}$; Bm = .625, or $\frac{5}{8}$; Bn = 1; and Bp = 2. Then the slope AoB will be $\frac{1}{\frac{5}{6}}$, or $\frac{6}{5}$; AmB = $\frac{1}{\frac{5}{8}}$, or $\frac{8}{5}$; AnB = $\frac{1}{1}$; and ApB = $\frac{1}{2}$.

In general, if r represents any difference of references, l the corresponding length of projection, and s the slope, we have

$$s = \frac{r}{l}.$$

If the unit be l, $s = r$.

If the unit be r, $s = \frac{1}{l}$.

But if $s = \frac{1}{l}$, then reciprocally, $l = \frac{1}{s}$.

A line is then known, when its projection, its slope (or its scale of slope), and the reference of *one point*, are given.

In Fig. 50, let AE be the line, of which the point A (ref. 9.4), and the slope $\frac{2}{9}$ are given.

$$\text{Then } l = \frac{1}{s} = \frac{1}{\frac{2}{9}} = \tfrac{9}{2} = 4.5.$$

Laying off, from the horizontal scale, 4.5 yds. from P to Q, the projection of a second point C is found, whose reference is 8.4. The scale of slope of the line can now be constructed.

If the scale of slope is given, with the projection of the line, and the reference of one of its points, the slope is found by the reciprocal of the preceding expression: thus,

$$s = \frac{1}{l} = \frac{1}{4.5} = \frac{1}{\frac{9}{2}} = \tfrac{2}{9}, \text{ or 2 upon 9.}$$

Problem V.

Knowing the projection of a point on a given right line; to find the reference of that point.

Let the line ND (Fig. 54) be given by its projection and by the references AP and BR of two of its points A and B. It is required to determine the reference MQ of the point M, whose projection Q is given. Let AP = 8; BR = 4.7, and PR = 10.5. Measure, on the horizontal scale, the length of PQ, and suppose it is found to be 7. Draw CM and GB, and by similar triangles we have $\frac{AC}{AG} = \frac{CM}{GB}$, or $\frac{AP-MQ}{AP-BR} = \frac{PQ}{PR}$, whence we deduce

$$MQ = AP - \frac{PQ}{PR}(AP - BR):$$

or, substituting the known numerical values in the second member,

$$MQ = 8 - \tfrac{7}{10.5}(8 - 4.7) = 5.8.$$

When the given point is N, and does not lie between the points A and B, the same result is reached by considering the triangles NFA and AGB; by which we have

$$\frac{AF}{AG} = \frac{NF}{GB}, \text{ or } \frac{NO - AP}{AP - BR} = \frac{OP}{PR}$$

whence we deduce

$$NO = AP + \frac{OP}{PR}(AP - BR).$$

Measuring OP as before, and finding it to be 3.1, we substitute the numerical values in the second member, and find

$$NO = 8 + .294 \times 3.3 = 9.29.$$

When the scale of slope is given, it is necessary only to find, from the subdivisions of the scale of slope, the distance of the point from either one of the principal divisions between which it lies; as in Fig. 55, which represents a scale of slope.

To find the reference of M; its distance measured from 5, is eight, and from 6, is two, of the subdivisions. Hence its reference is $5 + .8$, or $6 - .2$, which is 5.8 in either case. The point D, marked zero, is evidently the point where the given line pierces the plane of reference.

Problem VI.

Knowing the reference of a point on a given right line, to find the projection of that point.

Knowing the reference of the point M, which is supposed to lie between A and B (Fig. 54), it is required to determine the projection Q of that point; or, in other words, to calculate the length of PQ.

The similar triangles AGB and ACM give

$$\frac{CM}{GB} = \frac{AC}{AG}, \text{ or } \frac{PQ}{PR} = \frac{AP - MQ}{AP - BR}$$

whence we deduce

$$PQ = PR \times \frac{AP - MQ}{AP - BR}.$$

Or, substituting the numerical values,

$$PQ = 10.5 \times \frac{8 - 5.9}{8 - 4.7} = 6.99 +, \text{ or } 7.$$

For the known reference of T, the same formula will be obtained.

If the point were at N, we have

$$\frac{NF}{GB} = \frac{AF}{AG}, \text{ or } \frac{OP}{PR} = \frac{NO - AP}{AP - BR},$$

whence

$$OP = PR \times \frac{NO - AP}{AP - BR},$$

or, substituting the numerical values,

$$OP = 8\,\frac{9.29 - 8}{3.3} = 3.13.$$

If the scale of slope were known, the projection corresponding to the given reference can be taken from the scale, as in the last problem.

Note. In order to obtain the projection of the last point of the line above the plane of reference, or the point in which it pierces that plane, we have, from the triangles AGB and APD,

$$\frac{PD}{GB} = \frac{AP}{AG}, \text{ or } \frac{PD}{PR} = \frac{AP}{AP - BR}.$$

whence

$$PD = \frac{PR \times AP}{AP - BR}, = \frac{10.5 \times 8}{3.3}, = 25.45.$$

PROBLEM VII.

Through a given point to draw a line parallel to a given line.

Let AB (Fig. 56) be the given line, and C the given point. If the projection and scale of slope of the given line, and the projection of the point are given; draw through the given point a line parallel to the given line. It will be the projection of the required line. Suppose the reference of the given point to be 7.5. Find, on the scale of slope of the given line, the point whose reference is 7.5, and join it with the given point. Parallel to this last line draw lines through the divisions of the scale of slope of the given line. These parallels will be *horizontals*, and their intersections with the projection of the required line will mark its scale of slope; which, being the same as that of the given line, and their projections being parallel, the lines are parallel in space.

If the given line is known only by its projection and the references of two of its points, viz., A and B (refs. 8 and 5); take the length of projection between A and B, and lay it off in either direction from C on the required line. The references of the points so marked will be the difference of the references of A and B (3 in this case), added to or subtracted from the reference of C, viz.: 7.5 ± 3, = 10.5 or 4.5. Then the slope of the required line being the same for equal lengths of projection as that of the given line, and the projections parallel, the lines will be parallel in space.

The representation of a plane surface.

Any plane not parallel to the plane of reference will intersect it in a straight line, called *the trace* of the plane; and the angle

made by the oblique plane with the plane of reference will be measured by (that is, it will always be the same angle as) the angle formed by two lines, one in each plane, meeting the intersection of the planes at the same point of, and perpendicular to, that intersection. A line drawn in an oblique plane, perpendicular to the trace of the plane, will form, with the projection of the line, this measuring angle ; and the line so drawn in the oblique plane will be *its line of greatest descent*, or *its line of slope*.

A plane then is determined if the direction of its line of slope is known, and the scale of slope of that line, which is also evidently the scale of slope of the plane. To distinguish the scale of slope of a plane from that of a line, the former is represented by a *double line* (Fig. 57). If a plane is *horizontal* it is represented by the reference of any one of its points, as the references are the same for all of its points. If a plane is *vertical*, it is represented only by its trace on the plane of reference. The direction of the trace of a plane is known when that of its line of slope is known, and conversely, the latter is known if the former is given, since those two lines are always perpendicular to each other.

The projection of any point C (Fig. 57), situated in an oblique plane being given, the reference of that point can be determined by drawing C D perpendicular to the scale. C D will be a horizontal, and the reference of D read on the scale will be the required reference of the point C.

Problem VIII.

To pass a plane through three given points, and determine its scale of slope.

Let A, B, and C (Fig. 58) be the three points whose given references are respectively 12, 8.5, and 6. Join any two of the points, as A and C, and construct the scale of slope of the line AC. Find upon it the point H, whose reference is 8.5, and join it with B. This line HB will evidently be a horizontal of the required plane. Then FG, perpendicular to BH, will be a line of greatest descent of the plane, and upon it the scale of slope can be constructed. The point 8.5 of the scale is known, and by drawing through A the horizontal AK, parallel to BH,

the point 12 of the scale will be determined; or, by drawing the horizontal CD, the point 6 of the scale will be found, and the scale may then be constructed.

Problem IX.

Through a given point and a given line, to pass a plane, and determine its scale of slope.

Find upon the given line the point which has the same reference as that of the given point. Join it with the given point. This line will be a horizontal of the required plane. Parallel to that draw a line through any other point of the scale of the given line. This line will be another horizontal of the required plane. The scale of slope of the plane can then be constructed, as in Prob. VIII.

Problem X.

To pass a plane through a given point, parallel to a given plane, and determine its scale of slope.

Let BC, Fig. 59, be the scale of slope of the given plane, and A (5.7) be the given point. Since the planes are to be parallel, the lines of greatest descent of both will be parallel to each other. Therefore the scales of slope will be parallel. Since they also make the same angle with the plane of reference, their scales of slope will be graduated in divisions of the same length in each. Drawing through A a line parallel to BC, it will be the required line of slope, upon which, starting with the given reference 5.7, the scale may be constructed. If the reference of the given point is not fractional, the scale of slope can be laid off at once from the given scale.

Problem XI.

To determine the projection and scale of slope of the line of intersection of two given planes.

Let AB and CD (Fig. 60) be the scales of slope of the given planes. Take upon AB the points G and K at any convenient distance, and upon CD the points H and L, having the same

references respectively as G and K. Draw GO and KP perpendicular to AB, and HO and LP perpendicular to CD. These lines will be horizontals of the given planes, and their intersections O and P will evidently be points common to the two given planes. Joining O and P will give the required intersection. The scale of slope will in this case be found by dividing OP into five equal parts. The zero point of the line of intersection coincides with the intersection of the perpendiculars at A and C.

Problem XII.

To determine the projection and the reference of the point in which a given line intersects a given plane.

If any plane be passed through the given line, and the intersection of that plane with the given plane be found, it is evident that the given line will pierce the given plane at the point where it meets that intersection. A convenient plane to use for this problem is the plane which has the same scale of slope as the given line. Let AB (Fig. 61) be the scale of slope of the given plane, and FG that of the given line. Make FG the scale of slope of the auxiliary plane, and find its intersection CD with the given plane, by Prob. XI. The point O, in which the given line FG meets that intersection, is the point in which FG pierces the plane AB.

Problem XIII.

Fig. 62 shows the application of the preceding principles to a little problem of town-grading. The contour curves (dotted lines) show the natural form of the ground. The horizontal cutting planes are one yard apart. By means of these curves the references of all the street intersections are so assumed as to give the proper drainage slope to each street. These references are written at the proper points, and serve to determine the scale of slope of each line or portion of line. Through the points on these scales having the same references as the original contour curves (or any other points that the nature of the problem may dictate) the grade horizontals are drawn straight from point to point (broken and dotted lines).

From this illustration may be inferred the applicability of this method of delineation to problems in sewerage, grading of town-sites, park and lawn-grading, harbor-soundings, fortifications, embankments, &c., and in general, to all forms of ground where elevations and depressions are small when compared to horizontal extension.

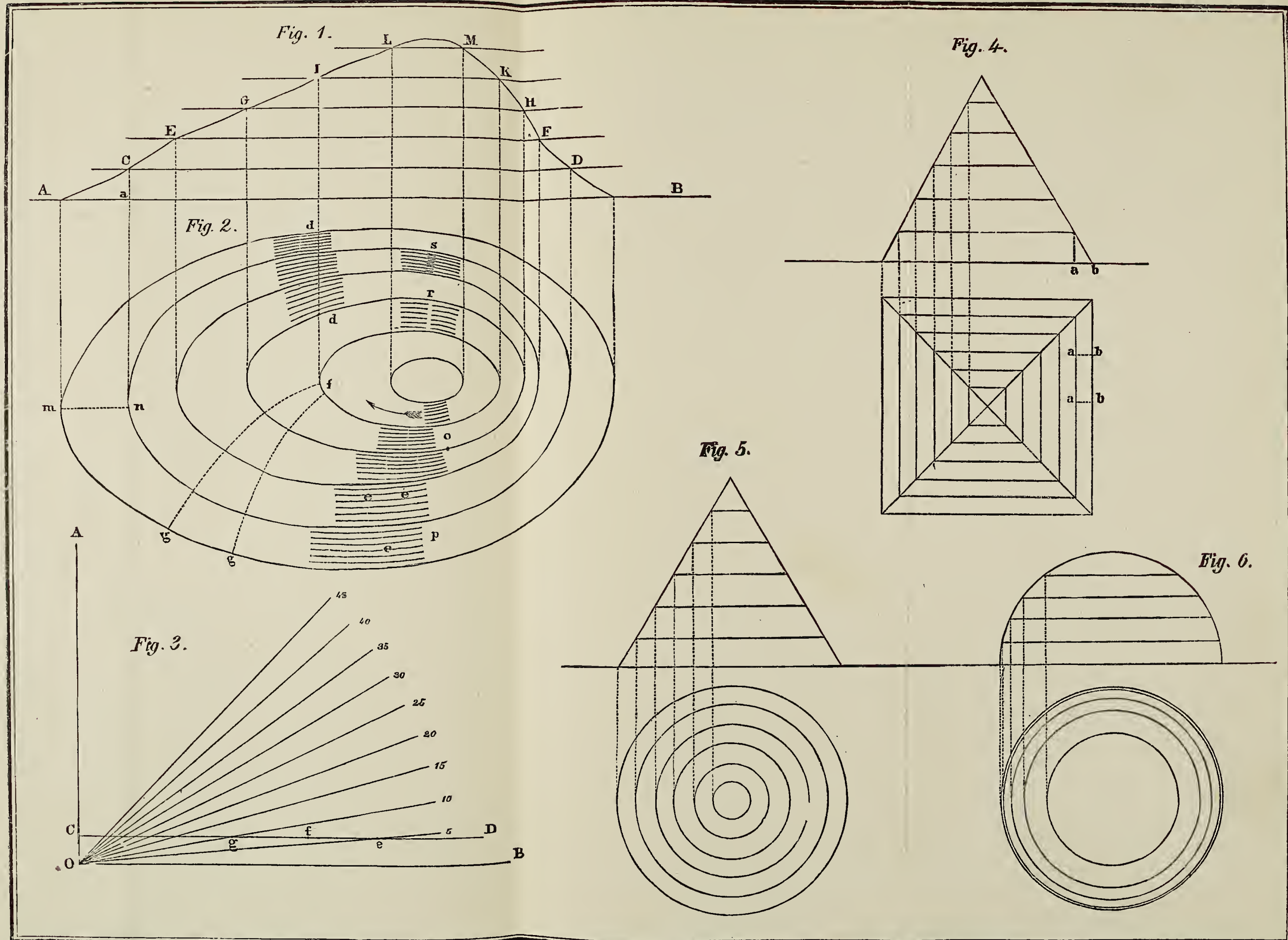
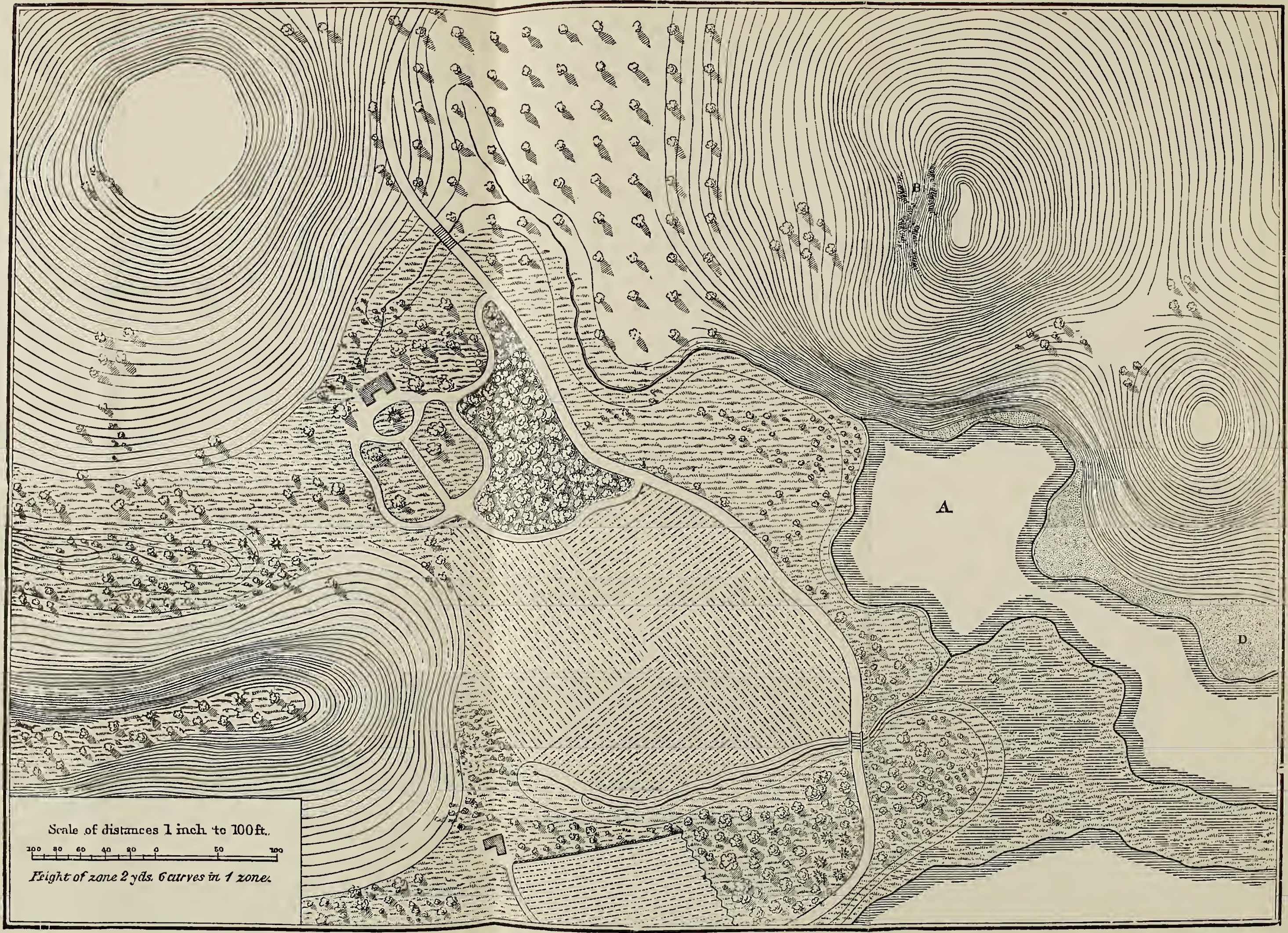


Fig. 1.
L M I K G H E F C D A a B
Fig. 2.
d s r d f o m n e e p e g g
Fig. 3.
A 45 40 35 30 25 20 15 10 5 C f D g e O B
Fig. 4.
a b a b a b
Fig. 5.
Fig. 6.



A Sketch in the horizontal System.

Fig. 8.

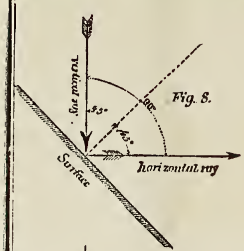


Fig. 9.
Lehman's scale of shade.

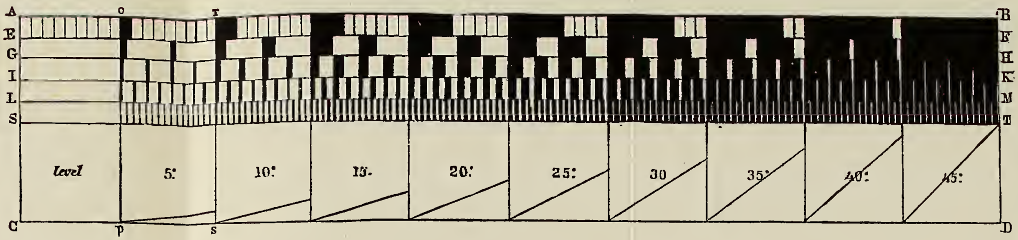


Fig. 10.

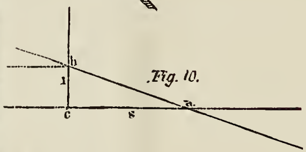


Fig. 11.

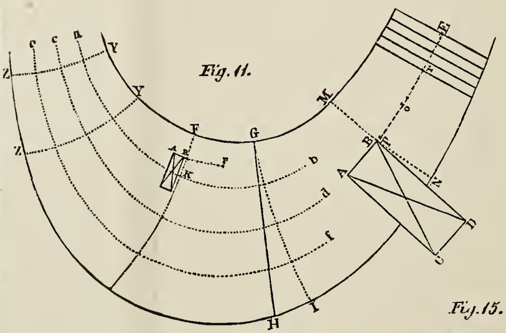


Fig. 12.

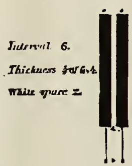


Fig. 13.
Scale of space for

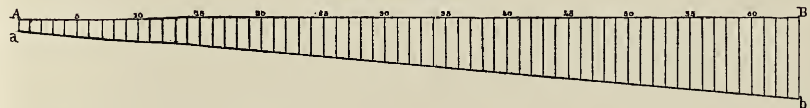


Fig. 14.
Tangent scale for height of zone 3 ft.

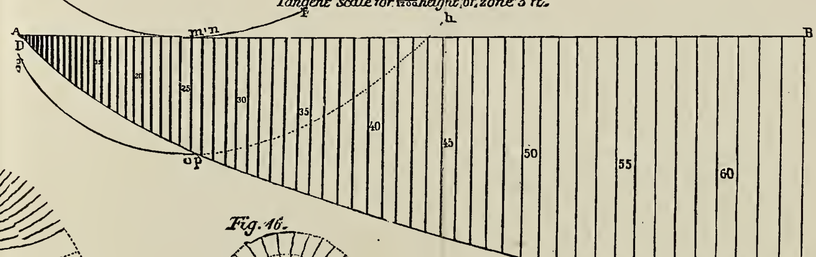


Fig. 15.

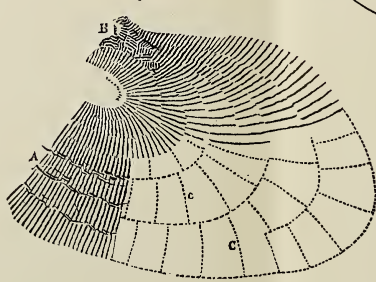
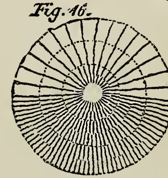


Fig. 16.



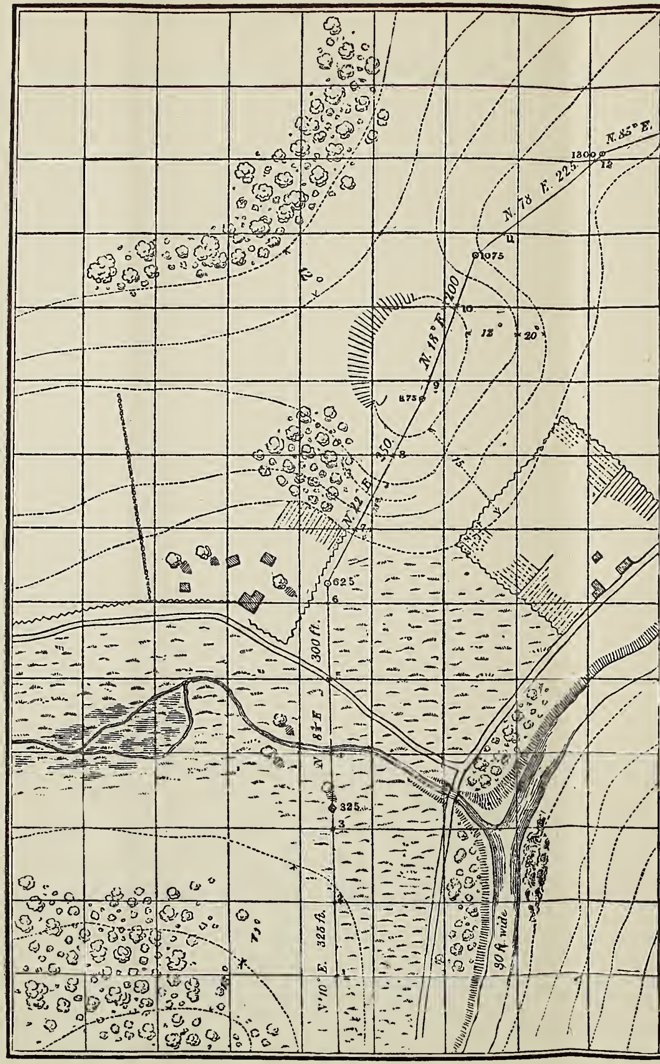
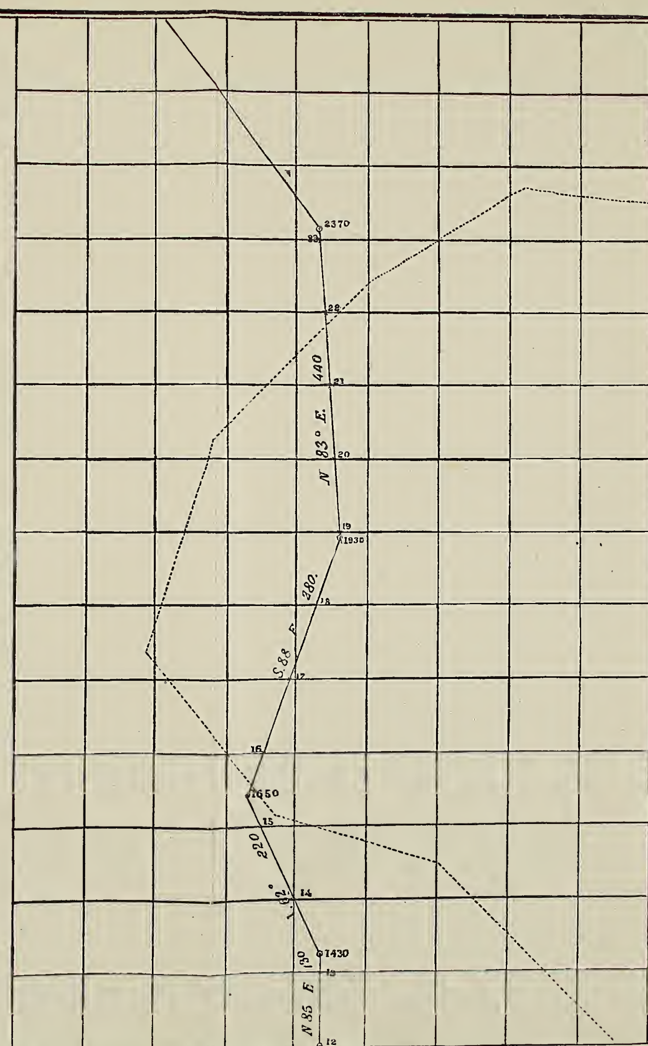


Fig. 16½
Field Sketch.



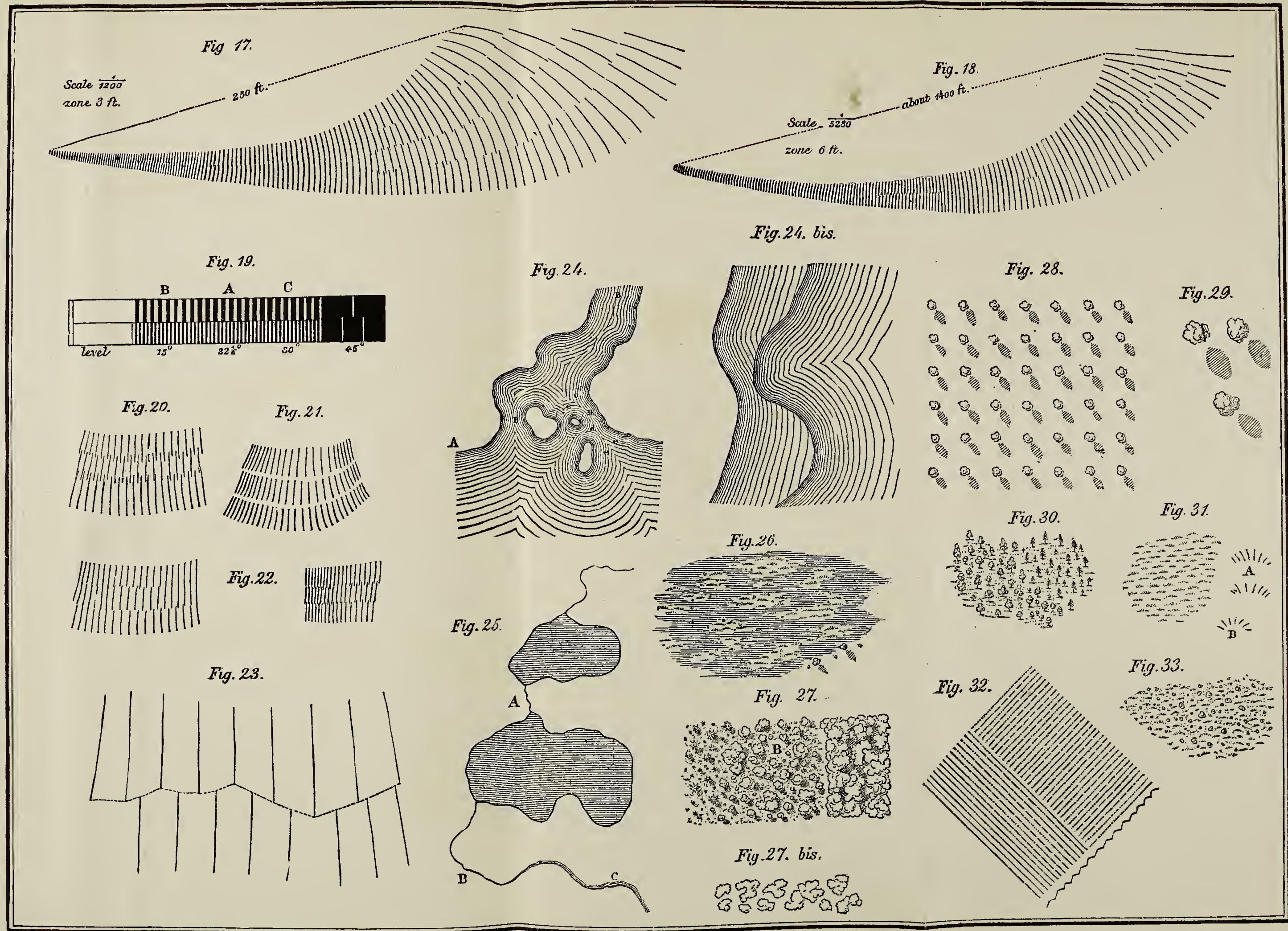
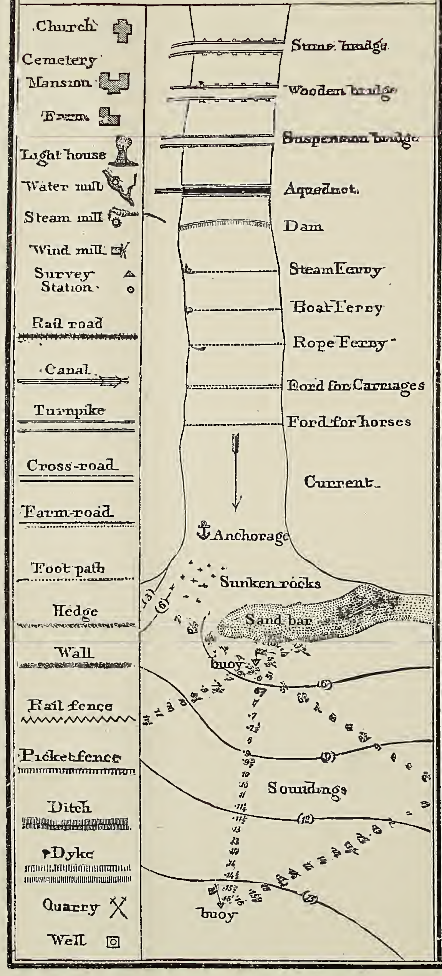


Fig 17.
Scale 1/1200
zone 3 ft.
250 ft.
Fig. 18.
Scale 1/5280
about 1400 ft.
zone 6 ft.
Fig. 19.
B
A
C
level
15°
22½°
30°
45°
Fig. 20.
Fig. 21.
Fig. 22.
Fig. 23.
Fig. 24.
A
B
Fig. 24. bis.
Fig. 25.
A
B
C
Fig. 26.
Fig. 27.
B
Fig. 27. bis.
Fig. 28.
Fig. 29.
Fig. 30.
Fig. 31.
A
B
Fig. 32.
Fig. 33.

Fig 34.


Church
Cemetery
Mansion
Farm
Light house
Water mill
Steam mill
Wind mill
Survey Station
Rail road
Canal
Turnpike
Cross-road
Farm-road
Foot path
Hedge
Wall
Rail fence
Picket fence
Ditch
Dyke
Quarry
Well
Stone bridge
Wooden bridge
Suspension bridge
Aqueduct
Dam
Steam Ferry
Boat Ferry
Rope Ferry
Ford for Carriages
Ford for horses
Current
Anchorage
Sunken rocks
Sand bar
buoy
Soundings
buoy

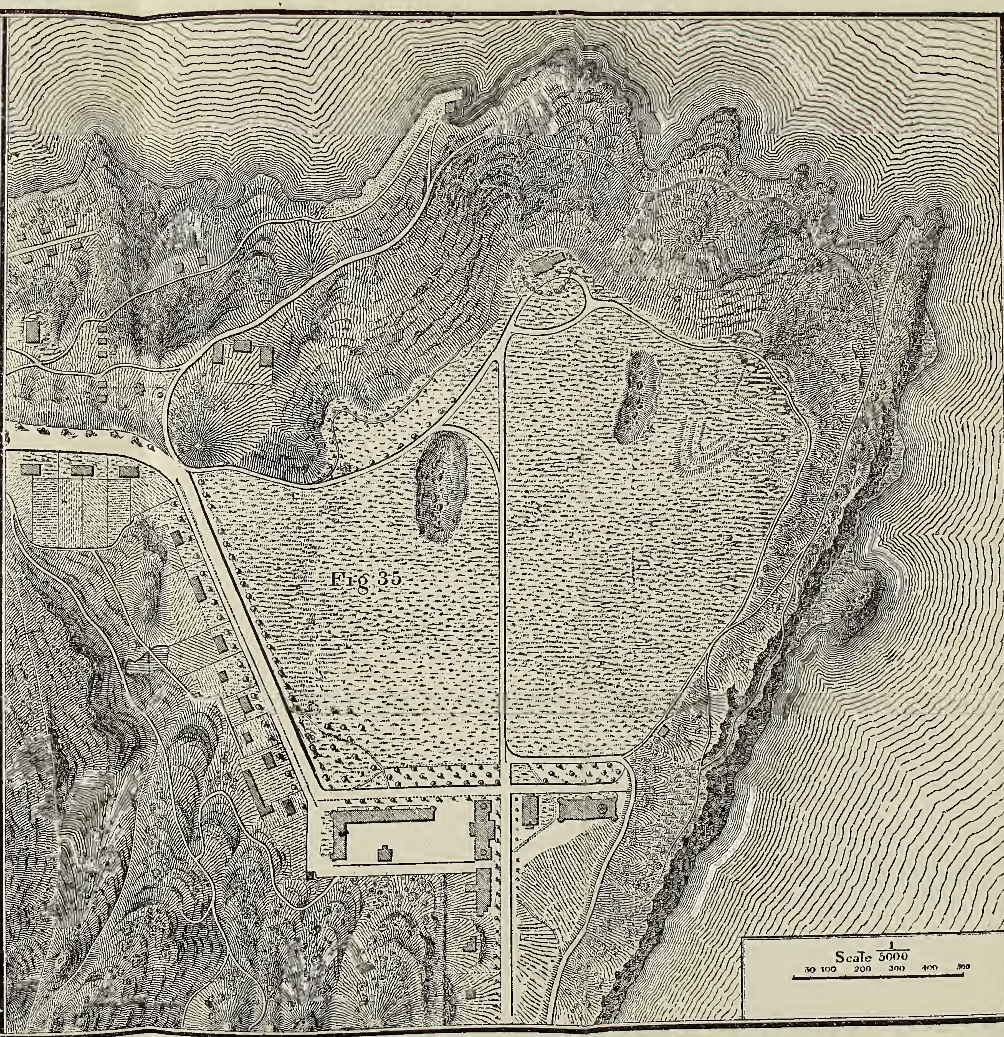

Scale $\frac{1}{5000}$
50 100 200 300 400 500

Fig 35

Fig. 37.

mbhpld
mbhpld

Fig. 38.

Scale of distances $\frac{1}{1000}$

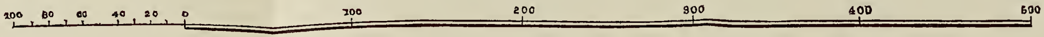


Fig. 39.

Scale of construction $\frac{1}{1000}$

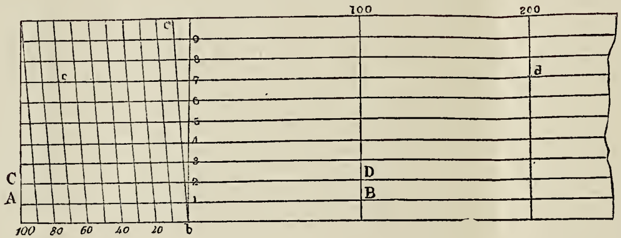


Fig. 40

Scale of construction, 24 inch. to 1 mile, $\frac{1}{2640}$

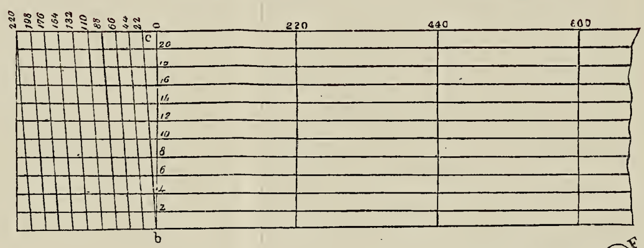


Fig. 41.

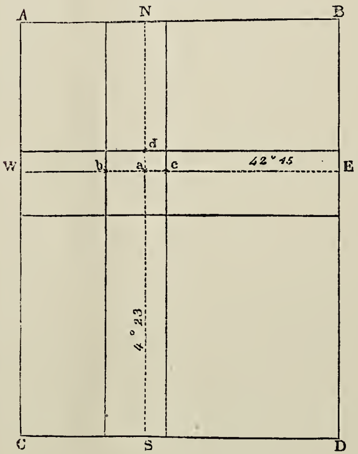


Fig. 42.

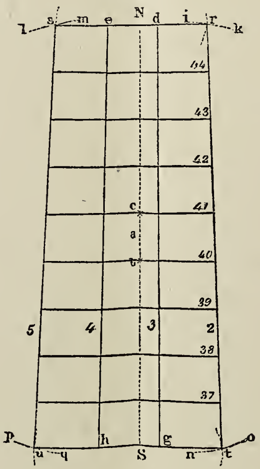


Fig. 44.

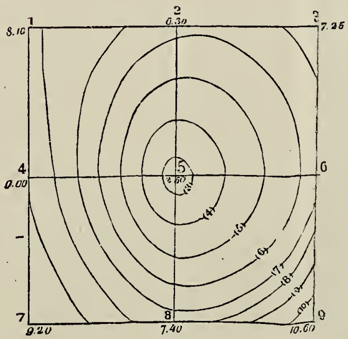
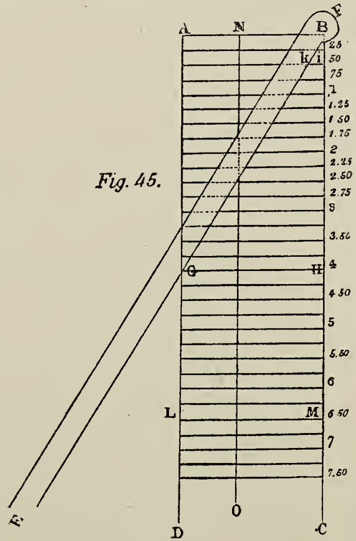


Fig. 45.



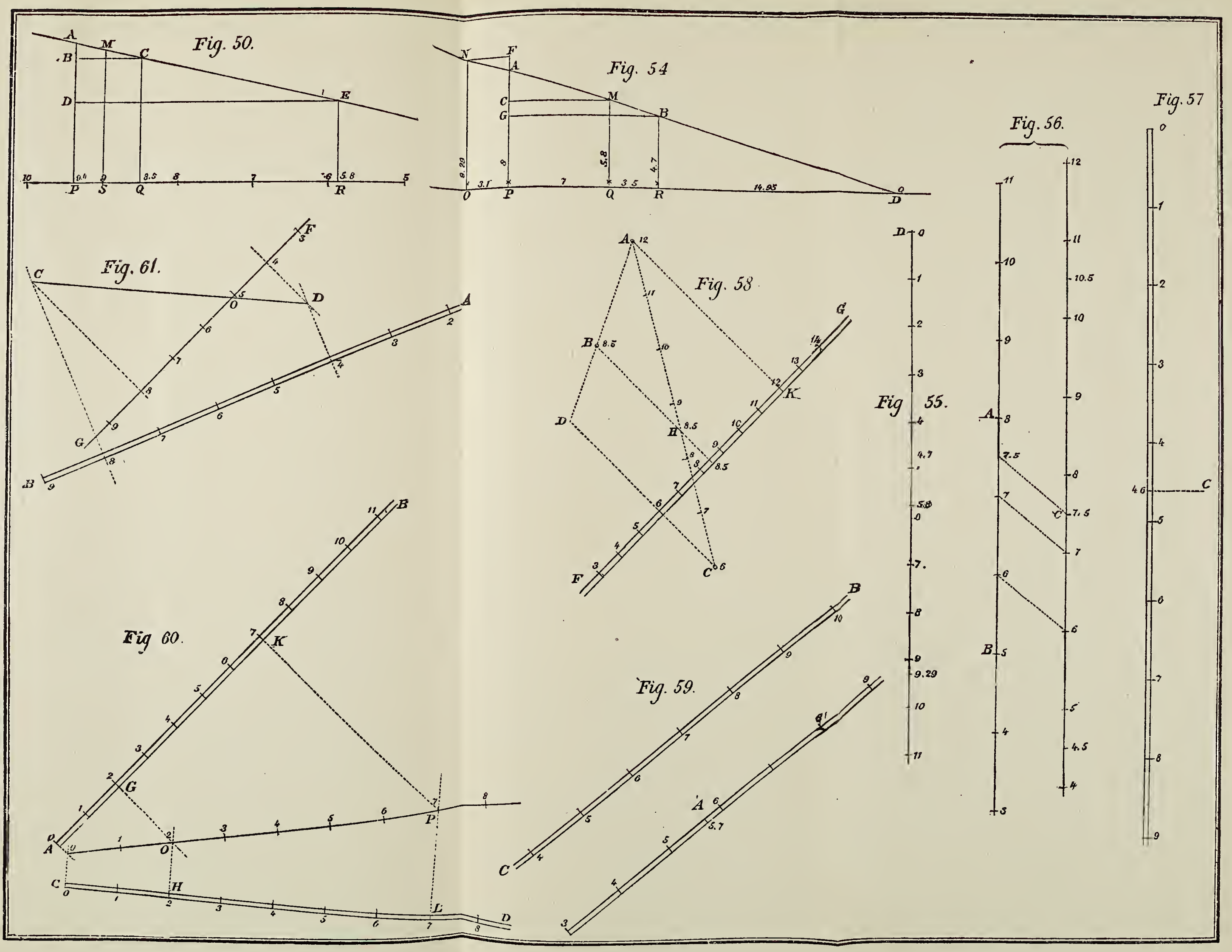
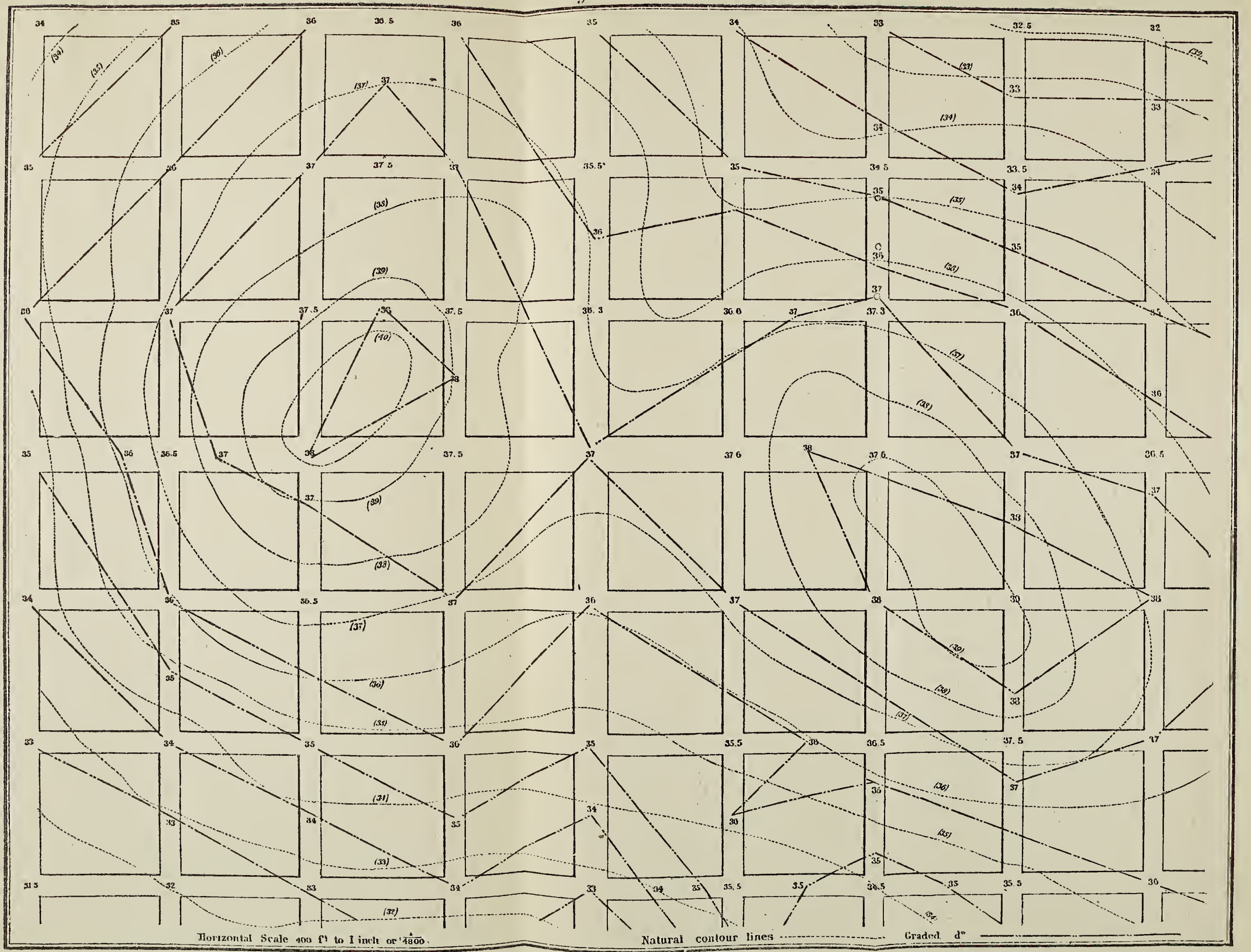


Fig. 50.
Fig. 54
Fig. 55.
Fig. 56.
Fig. 57
Fig. 58
Fig. 59.
Fig 60.
Fig. 61.

Fig. 62.



Horizontal Scale 400 ft to 1 inch or $\frac{1}{4800}$.
Natural contour lines
Graded d°